Multi-level Subjective Effects-based Assessment ☆

Johan Schubert
Division of Information Systems
Swedish Defence Research Agency
SE-164 90 Stockholm, Sweden
schubert@foi.se
http://www.foi.se/fusion/

***Abstract** - In this paper we develop a multi-level subjective effects-based assessment method. This method takes subjective assessments regarding activities and effects of a plan as inputs. From these assessments and a cross impact matrix that represents the impact between all elements of the plan we calculate combined assessments for all plan elements. For each activity (and effect) we calculate how much additional assessment value is needed to reach the assessment of the higher-level effect without its local assessment. The discrepancy between assessments received and assessments needed is an indication of relative performance of the activities. The method is based on belief functions and their combination under a generalization of the discounting operation.*

Keywords: Effects-Based Approach to Operations, Effects-Based Assessment, Subjective assessment, Dempster-Shafer theory, belief function, pseudo belief function, information fission, decomposition, inverse support function.

1 Introduction

In this paper we extend a previously developed subjective method for *Effects-Based Assessment* (EBA) [1−2] based on belief functions [3−9] and a *cross impact matrix* (CIM) [10−11] to handle assessments made at all effects levels within the *Effects-Based Planning* (EBP) [12] process.

A CIM can be used on the operational command level by the staff of a joint task force headquarter in an *Effects-Based Approach to Operations* [13] during planning, execution and assessment of an operation. The CIM consists of all *activities* (A), *supporting effects* (SE), *decisive conditions* (DC) and *military end state* (MES) of the plan. It is created by a broad working group which must assess how each activity impacts every other activity and supporting effect, how each supporting effect impacts every decisive condition (and possibly other supporting effects), and how every decisive condition impacts the military end state (and possibly other decisive conditions) [2, 14]. In this paper we use British concepts [15].

☆This work was supported by the FOI research project "Real-Time Simulation Supporting Effects-Based Planning", which is funded by the R&D programme of the Swedish Armed Forces.

The CIM can be used during assessment of the operation as it should contain the most current view of what impact all supporting effects have on the decisive conditions and what impact all decisive conditions have on the military end state.

Accepting human subjective assessments regarding the successful outcome of activities of the plan, we can use the impacts between plan elements as described by the CIM to calculate similar subjective assessments of all desired supporting effects, decisive conditions and the military end state. Using this methodology we get an early assessment of all plan elements during *Effects-Based Execution* (EBE) and may early on observe if activities and desired effects are developing according to plan. By observing the change over time of these subjective assessments of effects and conditions as assessments of activities are updated, we notice if trends are moving in the right direction as more activities are further executed.

We update the information fusion process to handle multiple-level inputs. In addition we introduce a new information fission process where combined fusion products are decomposed into its fission parts at the next lower level. This decomposition shows the assessments needed at the lower level to directly achieve the combined assessment at the next higher level without the assessment received at that higher level. The discrepancy between the assessments made and the assessments needed is measured and a high discrepancy can function as warning bell that some assessments may be wrong, or that some activities are not performing up to the average standard.

In Sec. 2 we describe the construction of a CIM. In Sec. 3 we develop an algorithm for assessment of plan elements using a CIM, and show how this may be used for subjective assessment of all desired effects. In Sec. 4 we introduce effects-level assessments, compare them with lower level assessments and push them upwards to discover consequences and downwards to discover assessments needed to achieve success. In Sec. 5 we evaluate discrepancies between assessments we have and assessments we need. In Sec. 6 we formulate an algorithm for multiple-level EBA. Finally, in Sec. 7 conclusions are drawn.

2 The plan an the CIM

The cross impact matrix will initially be created during the planning process. It should be created by a working group containing key subject matter experts as required by the type of operation planned. Before the CIM is constructed, a plan must be formed according to EBP. The plan consists of a military end state, decisive conditions, supporting effects and activities, Fig. 1.

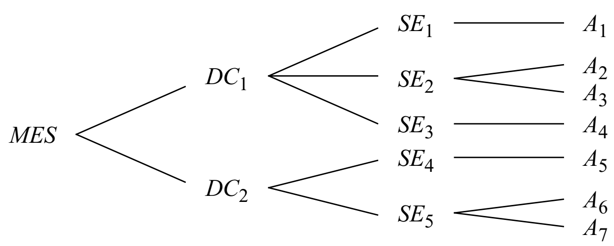

Fig. 1. Effects-based planning: *MES* = military end state, *DC* = decisive condition, *SE* = supporting effect, *A* = activity.

The working group will first need to enter all planned activities into the CIM, and it is important that all activities are well defined. They will then have to decide which positive or negative impact each activity will have on every other activity. It is important to note that even if activity A_1 has a positive impact on activity A_2, A_2 could have a negative impact on A_1. In the next step the working group must decide what impact all activities have on the supporting effects, what impact all supporting effects have on the decisive conditions and what impact the decisive conditions have on the military end state.

In Fig. 2 we will list these elements, except the military end state, to the left of the CIM and list the elements, including the military end state, above the CIM. The CIM consists of values ranging from -9 to +9, where -9 denotes large negative influence, 0 means no influence and +9 denotes high positive influence. For example, an impact value of +8, *i.e.*, "high positive influence", might be assigned between the activity of "securing an area" and the activity of "transporting through that area". How much element i influences element j is stored in the cell (i, j) in the CIM (for example activity A_2 influences activity A_4 in a positive way by a factor of +2, but A_4 influences A_2 in a negative way by a factor of -2). Its value is accessible through the function impact(i, j).

At the initial stage of the construction of the CIM we include the basic elements of the plan meaning that all activities should be performed and all supporting effects and decisive conditions should be reached.

	Military End State	DC_1	DC_2	SE_1	SE_2	A_1	A_2	A_3	A_4
DC_1	5	0	6	0	0	0	0	0	0
DC_2	8	6	0	0	0	0	0	0	0
SE_1	0	5	2	0	0	0	0	0	0
SE_2	0	5	8	0	0	0	0	0	0
A_1	0	0	0	3	3	0	4	-2	-3
A_2	0	0	0	3	-2	3	0	0	2
A_3	0	0	0	3	6	0	8	0	0
A_4	0	0	0	4	-2	-7	-2	0	0

Fig. 2. The CIM contains a military end state, decisive conditions, supporting effects and activities (dark gray cells always contain zeros). The CIM is the impact(i, j) matrix.

3 Assessment of plan elements

The CIM is a model of influence between elements of the plan. In assessment, our interest is on the impact between activities on the lowest level and supporting effects on the next level, and so forth. We receive subjective assessments regarding activities as user input. These are in the form of basic belief assignments (bbas) that express support for and against the success of that activity, encoded as AdP and ¬AdP, respectively.

3.1 Combining assessments

In this problem we have a simple frame of discernment

$$\Theta = \{\mathrm{AdP}, \neg\mathrm{AdP}\} \tag{1}$$

on each hierarchical level of the plan, where AdP means an *Adequate Plan*.

We have a set of n bbas each with three bodies of evidence, *i.e.*, $\{\{(\mathrm{AdP}, m_i(\mathrm{AdP})), (\neg\mathrm{AdP}, m_i(\neg\mathrm{AdP})), (\Theta, 1 - m_i(\mathrm{AdP}) - m_i(\neg\mathrm{AdP}))\}\}_{i=1}^{n}$, where, *e.g.*, $(\mathrm{AdP}, m_i(\mathrm{AdP}))$ is the first body of evidence of the ith bba giving support to AdP. Thus, for the ith bba we have,

$$m_i(A) = \begin{cases} m_i(\mathrm{AdP}), & A = \mathrm{AdP} \\ m_i(\neg\mathrm{AdP}), & A = \neg\mathrm{AdP} \\ 1 - m_i(\mathrm{AdP}) - m_i(\neg\mathrm{AdP}), & A = \Theta \end{cases}. \tag{2}$$

The CIM contains all information regarding the impact of each activity on all supporting effects. When the impact on a particular supporting effect SE_j is less than full we discount the bba m_i in relation to its degree of impact on SE_j

$$m_i^{\alpha_{ij}}(A) = \begin{cases} \alpha_{ij} m_i(\mathrm{AdP}), & A = \mathrm{AdP} \\ \alpha_{ij} m_i(\neg\mathrm{AdP}), & A = \neg\mathrm{AdP} \\ 1 - \alpha_{ij} m_i(\mathrm{AdP}) - \alpha_{ij} m_i(\neg\mathrm{AdP}), & A = \Theta \end{cases}. \tag{3}$$

For the sake of simplicity in combination of bbas, they are first transformed to commonalities using

$$Q(A) = \sum_{B \supseteq A} m(B). \tag{4}$$

Transforming all bbas $m_i^{\alpha_{ij}}(B)$ to commonalities $Q_i^{\alpha_{ij}}(A)$ using Eq. (4), we have

$$Q_i^{\alpha_{ij}}(A) = \begin{cases} 1 - \alpha_{ij} m_i(\neg\text{AdP}), & A = \text{AdP} \\ 1 - \alpha_{ij} m_i(\text{AdP}), & A = \neg\text{AdP} \\ 1 - \alpha_{ij} m_i(\text{AdP}) - \alpha_{ij} m_i(\neg\text{AdP}), & A = \Theta \end{cases} . \quad (5)$$

Let us now combine all commonalities $Q_i^{\alpha_{ij}}$, using Dempster's rule for commonalities. We get

$$Q_{\oplus\{m_i^{\alpha_{ij}}\}_{i=1}^n}(A) = K \prod_{i=1}^n Q_i^{\alpha_{ij}}(A)$$

$$= \begin{cases} K \prod_{i=1}^n [1 - \alpha_{ij} m_i(\neg\text{AdP})], & A = \text{AdP} \\ K \prod_{i=1}^n [1 - \alpha_{ij} m_i(\text{AdP})], & A = \neg\text{AdP} \\ K \prod_{i=1}^n [1 - \alpha_{ij} m_i(\text{AdP}) - \alpha_{ij} m_i(\neg\text{AdP})], & A = \Theta \end{cases} \quad (6)$$

where K is a normalizing constant and $n = |\{\alpha_{ij}\}_j|$.

Commonalities can be transformed back to bbas using

$$m(A) = \sum_{B \supseteq A} (-1)^{|B - A|} Q(B) . \quad (7)$$

Transforming back from commonality to bba, we get

$$m_{\oplus\{m_i^{\alpha_{ij}}\}_{i=1}^n}(A) =$$

$$= \begin{cases} K\left\{ \prod_{i=1}^n [1 - \alpha_{ij} m_i(\neg\text{AdP})] - \prod_{i=1}^n [1 - \alpha_{ij} m_i(\text{AdP}) - \alpha_{ij} m_i(\neg\text{AdP})] \right\}, & A = \text{AdP} \\ K\left\{ \prod_{i=1}^n [1 - \alpha_{ij} m_i(\text{AdP})] - \prod_{i=1}^n [1 - \alpha_{ij} m_i(\text{AdP}) - \alpha_{ij} m_i(\neg\text{AdP})] \right\}, & A = \neg\text{AdP} \\ K \prod_{i=1}^n [1 - \alpha_{ij} m_i(\text{AdP}) - \alpha_{ij} m_i(\neg\text{AdP})], & A = \Theta \end{cases} \quad (8)$$

where

$$\begin{aligned} K^{-1} = & \prod_{i=1}^n [1 - \alpha_{ij} m_i(\neg\text{AdP})] \\ & - \prod_{i=1}^n [1 - \alpha_{ij} m_i(\text{AdP}) - \alpha_{ij} m_i(\neg\text{AdP})] \\ & + \prod_{i=1}^n [1 - \alpha_{ij} m_i(\text{AdP})] \\ & - \prod_{i=1}^n [1 - \alpha_{ij} m_i(\text{AdP}) - \alpha_{ij} m_i(\neg\text{AdP})] \\ & + \prod_{i=1}^n [1 - \alpha_{ij} m_i(\text{AdP}) - \alpha_{ij} m_i(\neg\text{AdP})] \\ = & \prod_{i=1}^n [1 - \alpha_{ij} m_i(\neg\text{AdP})] \\ & - \prod_{i=1}^n [1 - \alpha_{ij} m_i(\text{AdP}) - \alpha_{ij} m_i(\neg\text{AdP})] \\ & + \prod_{i=1}^n [1 - \alpha_{ij} m_i(\text{AdP})]. \end{aligned} \quad (9)$$

Thus, Eq. (8) becomes the subjective assessment of SE_j as calculated using the subjective input assessments of all activities A_i that impact upon SE_j.

What is calculated for supporting effects from subjective assessment of activities can in a second phase be calculated for decisive conditions using the newly calculated assessments of supporting effects. In the same way we can calculate the subjective assessment of the military end state from the assessment of decisive conditions.

3.2 Combining assessments regarding plan elements using the CIM.

At the activities level we have a frame of discernment

$$\Theta_A = \{\text{AdP}, \neg\text{AdP}\} . \quad (10)$$

In order to map this onto the problem of combining assessments, Sec. 3.1, we must first generalize the discounting operation.

The discounting operation was introduced to handle the case when the source of some piece of evidence is lacking in credibility [6]. The credibility of the source, $0 < \alpha < 1$, also became the credibility of the piece of evidence. The situation was handled by discounting each supported proposition other than Θ with the credibility α and by adding the discounted mass to Θ;

$$m^{\alpha}(A) = \begin{cases} \alpha m(A), & A \neq \Theta \\ 1 - \alpha + \alpha m(\Theta), & A = \Theta \end{cases} . \quad (11)$$

In [1] we generalized the discounting operation by allowing the credibility to take values in the interval $-1 \leq \alpha < 1$.

Definition 1. Let $m: 2^{\Theta} \to [0, 1]$ be a bba where $-1 \leq \alpha < 1$. Then

$$m^{\alpha}(A) = \begin{cases} \alpha m(A), & A \neq \Theta \\ 1 - \alpha + \alpha m(\Theta), & A = \Theta \end{cases} . \quad (12)$$

is a generalized discounting of m where $m^{\alpha}(A)$ is an inverse simple support function (ISSF) whenever $\alpha < 0$.

Definition 2. An inverse simple support function (ISSF) on a frame of discernment Θ is a function $m:2^{\Theta} \to (-\infty, \infty)$ characterized by a weight $w \in (1, \infty)$ and a focal element $A \subseteq \Theta$, such that $m(\Theta) = w$, $m(A) = 1 - w$ and $m(X) = 0$ when $X \notin \{A, \Theta\}$.

Let us recall the meaning of simple support functions (SSFs) and ISSFs, [16]: An SSF $m_1(\mathrm{A}) \in [0, 1]$ represents a state of belief that "You have some reason to believe that the actual world is in A (and nothing more)". An ISSF $m_2(\mathrm{A}) \in (-\infty, 0)$, on the other hand, represents a state of belief that "You have some reason *not* to believe that the actual world is in A".

3.2.1 Assessment of supporting effects (SE)

Before combining the mass functions we discount them using the impact values of the CIM. This ensures that each activity influences the supporting effect to its proper degree.

For SE_j and A_i we have

$$m_{A_i}^{\alpha_{A_iSE_j}}(X) = \begin{cases} \alpha_{A_iSE_j} m_{A_i}(\mathrm{AdP}), & X = \mathrm{AdP} \\ \alpha_{A_iSE_j} m_{A_i}(\neg\mathrm{AdP}), & X = \neg\mathrm{AdP} \\ 1 - \alpha_{A_iSE_j} m_{A_i}(\mathrm{AdP}) - \alpha_{A_iSE_j} m_{A_i}(\neg\mathrm{AdP}), & X = \Theta \end{cases} \quad (13)$$

where the discounting factor is defined as

$$\alpha_{A_iSE_j} \triangleq \frac{\mathrm{impact}(A_i, SE_j)}{10}. \quad (14)$$

This is a generalization of the discounting operator where discounting factors may assume values less than 0, *i.e.*, $\alpha_{A_iSE_j} = \{-0.9, -0.8, -0.7, \ldots, 0.9\}$.

We combine all bbas on the activities level and bring the result to the supporting effects level. At the supporting effects level we have a similar frame of discernment,

$$\Theta_{SE} = \{\mathrm{AdP}, \neg\mathrm{AdP}\}. \quad (15)$$

Using Eq. (13) and Eq. (14) we define

$$\begin{aligned} m_{SE_j}(\mathrm{AdP}) &\triangleq m_{\oplus\{m_{A_i}^{\alpha_{A_iSE_j}}\}_{i=1}^{n}}(\mathrm{AdP}), \\ m_{SE_j}(\neg\mathrm{AdP}) &\triangleq m_{\oplus\{m_{A_i}^{\alpha_{A_iSE_j}}\}_{i=1}^{n}}(\neg\mathrm{AdP}), \\ m_{SE_j}(\Theta) &\triangleq m_{\oplus\{m_{A_i}^{\alpha_{A_iSE_j}}\}_{i=1}^{n}}(\Theta), \end{aligned} \quad (16)$$

which can be calculated by Eq. (8). Here, any $m_{SE_j}(\mathrm{AdP}), m_{SE_j}(\neg\mathrm{AdP}), m_{SE_j}(\Theta)$ may be < 0. When this is the case, m_{SE_j} is called a *pseudo belief function* [16].

3.2.2 Assessment of Decisive Conditions (DC), and the military end state (MES)

We have identical frames at the decisive condition level and the military end state level,

$$\Theta_{DC} = \{\mathrm{AdP}, \neg\mathrm{AdP}\} \quad (17)$$

and

$$\Theta_{MES} = \{\mathrm{AdP}, \neg\mathrm{AdP}\}. \quad (18)$$

We may calculate support for decisive conditions (DC) by substitution of SE_j with DC_j and of A_i with SE_i in Eq. (16). In the same way, we may calculate support for the military end state (MES) by substitution of SE_j with MES and of A_i with DC_i in Eq. (16). Doing this allows us to calculate support in all effects at every level step-by-step starting from the assessments of all A_i.

4 Effects level assessments

We can have data inputs directly at any node at the effects level by m_{SE_j}, m_{DC_j} and even m_{MES}. This may be an assessment by a commander. The inputs can be handled in two different ways, either as new information regarding the adequacy of the plan to be combined with assessments from lower levels, or as an assessment overriding the assessments made at lower levels. The combined assessments at say SE_j can be pushed upwards to discover the consequences at higher levels, and pushed downwards to discover what we would need to achieve at the lower levels to directly achieve the current support for SE_j.

4.1 Combined assessments

4.1.1 Information fusion sweep (bottom-up)

From any level and node, *e.g.*, SE_j, we may combine all commonalities to find the consequences for this node, and effect nodes at higher levels.

We have new commonalities

$$Q_{m_{SE_j} \oplus \{m_{A_i}^{\alpha_{A_iSE_j}}\}_{i=1}^{n}}(X) = KQ_{SE_j}(X) \prod_{i=1}^{n} Q_{A_i}^{\alpha_{A_iSE_j}}(X)$$

$$= \begin{cases} K[1 - m_{SE_j}(\neg\mathrm{AdP})] \times \prod_{i=1}^{n} [1 - \alpha_{A_iSE_j} m_{A_i}(\neg\mathrm{AdP})], & X = \mathrm{AdP} \\ K[1 - m_{SE_j}(\mathrm{AdP})] \times \prod_{i=1}^{n} [1 - \alpha_{A_iSE_j} m_{A_i}(\mathrm{AdP})], & X = \neg\mathrm{AdP} \\ K[1 - m_{SE_j}(\mathrm{AdP}) - m_{SE_j}(\neg\mathrm{AdP})] \times \prod_{i=1}^{n} [1 - \alpha_{A_iSE_j} m_{A_i}(\mathrm{AdP}) - \alpha_{A_iSE_j} m_{A_i}(\neg\mathrm{AdP})], & X = \Theta \end{cases} \quad (19)$$

in the case of a commander contributing an opinion m_{SE_j} to be combined with field views from the activities m_{A_i}, where

$$m_{m_{SE_j} \oplus \{m_{A_i}^{\alpha_{A_iSE_j}}\}_{i=1}^{n}}(X) = \begin{cases} Q_{m_{SE_j} \oplus \{m_{A_i}^{\alpha_{A_iSE_j}}\}_{i=1}^{n}}(\mathrm{AdP}) - Q_{m_{SE_j} \oplus \{m_{A_i}^{\alpha_{A_iSE_j}}\}_{i=1}^{n}}(\Theta), & X = \mathrm{AdP} \\ Q_{m_{SE_j} \oplus \{m_{A_i}^{\alpha_{A_iSE_j}}\}_{i=1}^{n}}(\neg\mathrm{AdP}) - Q_{m_{SE_j} \oplus \{m_{A_i}^{\alpha_{A_iSE_j}}\}_{i=1}^{n}}(\Theta), & X = \neg\mathrm{AdP} \\ Q_{m_{SE_j} \oplus \{m_{A_i}^{\alpha_{A_iSE_j}}\}_{i=1}^{n}}(\Theta), & X = \Theta. \end{cases} \tag{20}$$

This process can be repeated for all nodes on the *SE* level, followed by all nodes at the *DC* level, and finally once at the *MES* level.

In Fig. 3 an example with field views of the activities is fused. In Fig. 4 we add a commander's view regarding m_{SE_3}. Observe the difference in the fused output for m_{SE_3} marked **SE 3** in Fig. 3 and $m_{m_{SE_3} \oplus \{m_{A_i}^{\alpha_{A_iSE_3}}\}}$ marked **SE 3** in Fig. 4.

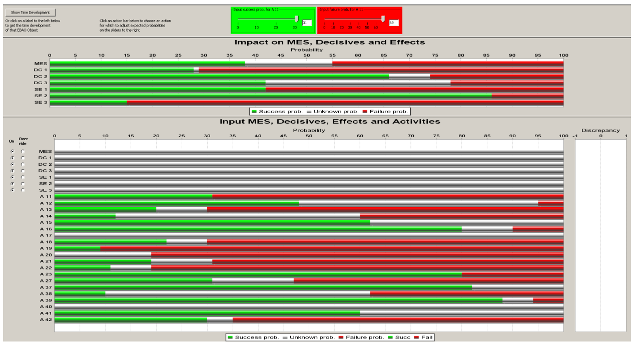

Fig. 3. Information fusion sweep (bottom-up) of field views from the activities m_{A_i}.

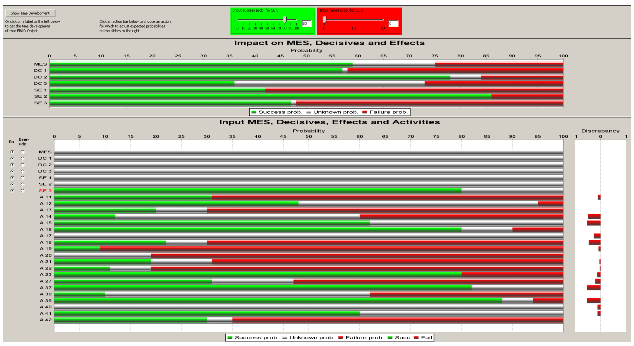

Fig. 4. Information fusion sweep (bottom-up) adding a commander's view.

4.1.2 *Information fission sweep (top-down)*

For the case of combining assessment we state that the combination of all commonalities for a node SE_j, $Q_{SE_j}(X)$, received at this level as well as all commonalties received from the activities level is *set* equal to new commonalities that we would need in order to achieve the same support at SE_j, without $Q_{SE_j}(X)$. We have,

$$Q_{m_{SE_j} \oplus \{m_{A_i}^{\alpha_{A_iSE_j}}\}_{i=1}^{n}}(\neg\mathrm{AdP}) = Q^{*\,SE_j}_{\oplus \{m_{A_i}^{*\,SE_j}\}_{i=1}^{n}}(\neg\mathrm{AdP}) \tag{21}$$

where the left hand side (LHS) is the combination of the commonality at SE_j with all discounted commonalities from the activities A_i for all i, while the right hand side (RHS) is the combined commonalities of the sought-after bbas we would have had to get the same support at the SE_j node without m_{SE_j}.

The LHS can be rewritten as follows, by simultaneous division and multiplication with the commonalities from the activities A_i for all i, without $Q_{SE_j}(X)$. We have,

$$\frac{Q_{m_{SE_j} \oplus \{m_{A_i}^{\alpha_{A_iSE_j}}\}_{i=1}^{n}}(\neg\mathrm{AdP})}{Q_{\oplus \{m_{A_i}^{\alpha_{A_iSE_j}}\}_{i=1}^{n}}(\neg\mathrm{AdP})} Q_{\oplus \{m_{A_i}^{\alpha_{A_iSE_j}}\}_{i=1}^{n}}(\neg\mathrm{AdP}) = Q^{*\,SE_j}_{\oplus \{m_{A_i}^{*\,SE_j}\}_{i=1}^{n}}(\neg\mathrm{AdP}). \tag{22}$$

This can be rewritten as

$$\frac{Q^{\alpha_{A_iSE_j}}_{m_{SE_j} \oplus \{m_{A_i}^{\alpha_{A_iSE_j}}\}_{i=1}^{n}}(\neg\mathrm{AdP})}{Q^{\alpha_{A_iSE_j}}_{\oplus \{m_{A_i}^{\alpha_{A_iSE_j}}\}_{i=1}^{n}}(\neg\mathrm{AdP})} \prod_{i=1}^{n} \left[1 - \alpha_{A_iSE_j} m_{A_i}(\mathrm{AdP})\right] = \prod_{i=1}^{n} \left[1 - \alpha_{A_iSE_j} m_{A_i}^{*\,SE_j}(\mathrm{AdP})\right] \tag{23}$$

At this stage we have to make an assumption. We assume that each term in the product at the RHS is scaled by the same factor. When this is the case we can deduce the underling bba at the activity level, needed to cause the support of *SE* had m_{SE_j} *not* been present. The factor becomes,

$$\sqrt[n]{\frac{Q^{\alpha_{A_iSE_j}}_{\oplus \{m_{A_i}^{\alpha_{ij}}\}_{i=1}^{n}}(\neg\mathrm{AdP})}{Q^{\alpha_{A_iSE_j}}_{m_{SE_j} \oplus \{m_{A_i}^{\alpha_{ij}}\}_{i=1}^{n}}(\neg\mathrm{AdP})}}, \tag{24}$$

where we have an nth root with $n = |\{\alpha_{ij}\}_j|$ since the factor comes in n times in the product.

With this assumption in place, we get

$$\sqrt[n]{\frac{Q^{\alpha_{A_iSE_j}}_{m_{SE_j} \oplus \{m_{A_i}^{\alpha_{ij}}\}_{i=1}^{n}}(\neg\mathrm{AdP})}{Q^{\alpha_{A_iSE_j}}_{\oplus \{m_{A_i}^{\alpha_{ij}}\}_{i=1}^{n}}(\neg\mathrm{AdP})}} \left[1 - \alpha_{A_iSE_j} m_{A_i}(\mathrm{AdP})\right] = 1 - \alpha_{A_iSE_j} m_{A_i}^{*\,SE_j}(\mathrm{AdP}). \tag{25}$$

This can be rewritten as

$$m_{A_i}^{*\,SE_j}(\mathrm{AdP}) = \frac{1 - \sqrt[n]{\dfrac{Q^{\alpha_{A_iSE_j}}_{m_{SE_j} \oplus \{m^{\alpha_{ij}}_{A_i}\}_{i=1}^n}(\neg\mathrm{AdP})}{Q^{\alpha_{A_iSE_j}}_{\oplus\{m^{\alpha_{ij}}_{A_i}\}_{i=1}^n}(\neg\mathrm{AdP})}\left[1 - \alpha_{A_iSE_j} m_{A_i}(\mathrm{AdP})\right]}}{\alpha_{A_iSE_j}}$$

$$= \frac{1 - \sqrt[n]{1 - m_{SE_j}(\neg\mathrm{AdP})\left[1 - \alpha_{A_iSE_j} m_{A_i}(\mathrm{AdP})\right]}}{\alpha_{A_iSE_j}}, \tag{26}$$

whenever $\alpha_{A_iSE_j} \neq 0$, otherwise $m_{A_i}^{*\,SE_j}$ is undefined.

In a similar way we can derive,

$$m_{A_i}^{*\,SE_j}(\neg\mathrm{AdP}) = \frac{1 - \sqrt[n]{\dfrac{Q^{\alpha_{A_iSE_j}}_{m_{SE_j} \oplus \{m^{\alpha_{ij}}_{A_i}\}_{i=1}^n}(\mathrm{AdP})}{Q^{\alpha_{A_iSE_j}}_{\oplus\{m^{\alpha_{ij}}_{A_i}\}_{i=1}^n}(\mathrm{AdP})}\left[1 - \alpha_{A_iSE_j} m_{A_i}(\neg\mathrm{AdP})\right]}}{\alpha_{A_iSE_j}}$$

$$= \frac{1 - \sqrt[n]{1 - m_{SE_j}(\mathrm{AdP})\left[1 - \alpha_{A_iSE_j} m_{A_i}(\neg\mathrm{AdP})\right]}}{\alpha_{A_iSE_j}} \tag{27}$$

and

$$m_{A_i}^{*\,SE_j}(\Theta) = 1 - m_{A_i}^{*\,SE_j}(\mathrm{AdP}) - m_{A_i}^{*\,SE_j}(\neg\mathrm{AdP}). \tag{28}$$

Note, that the $m_{DC_i}^{*\,MES}$, $m_{SE_i}^{*\,DC_j}$, $m_{A_i}^{*\,SE_j}$ calculated during the top-down information fission sweep are bodies of evidence regarding different propositions than the m_{DC_i}, m_{SE_i}, m_{A_i} calculated during the bottom-up information fusion sweep. The information fusion sweep calculates assessment that we have, while the information fission sweep calculate which assessment we need to fulfill without the higher multiple-level assessments received at the effects nodes.

When there are several multiple-level assessments on the *SE* level, or one or more at the *DC* level, or one at the *MES* level, then we receive a family of belief functions (or *pseudo belief functions* [16]) $m_{A_i}^{*\,SE_j}$, one from each SE_j where there are multiple-level assessments. Similarly on the *SE* level if we have several multiple-level assessments on the *DC* level or one at the *MES* level, and also for the *DC* level if there is an assessment of *MES*.

It is possible that $m_{A_i}^{*\,SE_j}$ Eq. (26), Eq. (27), Eq. (28) is a pseudo belief function where some bbns are negative. When this is the case we may first decompose $m_{A_i}^{*\,SE_j}$ into its two separate components (SSFs or ISSFs) for and against the plan $\{\mathrm{AdP}, \neg\mathrm{AdP}\}$ using the decomposition introduced by Smets [16], *i.e.*, one supporting AdP and Θ, and the other supporting ¬AdP and Θ.

We have,

$$\begin{aligned} m_{m_{A_i}}^{*\,SE_j^{\circ}}(X) &= 1 - \prod_{Y \supseteq X} Q_{m_{A_i}}^{*\,SE_j}(Y)^{(-1)^{|Y|-|X|+1}}, \\ m_{m_{A_i}}^{*\,SE_j^{\circ}}(\Theta) &= 1 - m_{m_{A_i}}^{*\,SE_j^{\circ}}(X), \end{aligned} \tag{29}$$

where $\circ \in \{+, -\}$ for $X \in \{\mathrm{AdP}, \neg\mathrm{AdP}\}$, respectively.

If it turns out that $m_{m_{A_i}}^{*\,SE_j^{\circ}}(X) < 0$ (*i.e.*, an ISSF) we may instead calculate the degree to which we do *not* believe in that proposition. This is something more easily understood.

We have,

$$\begin{aligned} m_{A_i}^{*\,SE_j^{\circ}\ not}(not \text{ believe in } X) &= 1 - \frac{1}{1 - m_{A_i}^{*\,SE_j}(X)}, \\ m_{A_i}^{*\,SE_j^{\circ}\ not}(\Theta) &= 1 - m_{A_i}^{*\,SE_j^{\circ}\ not}(not \text{ believe in } X), \end{aligned} \tag{30}$$

where $m_{A_i}^{*\,SE_j^{\circ}\ not}(not \text{ believe in } X) > 0$ when $m_{A_i}^{*\,SE_j}(X) < 0$, and $\circ \in \{+, -\}$.

4.2 Overriding assessments

In the case with overriding high-level assessments that overrides the field view assessments from lower levels Eq. (19) is substituted by

$$Q_{SE_j}^{\#}(X) = \begin{cases} [1 - m_{SE_j}(\neg\mathrm{AdP})], & X = \mathrm{AdP} \\ [1 - m_{SE_j}(\mathrm{AdP})], & X = \neg\mathrm{AdP} \\ [1 - m_{SE_j}(\mathrm{AdP}) - m_{SE_j}(\neg\mathrm{AdP})], & X = \Theta. \end{cases} \tag{31}$$

Here, the #-sign indicates that $Q_{SE_j}^{\#}$ is an overriding assessment, different from the non-overriding assessment Q_{SE_j} that was assigned at this level and combined with the commonalities from the activities A_i. in the previous subsection.

4.2.1 Information fusion sweep

There is no information process leading up towards a node where there is an overriding assessment received. The overriding assessment is instead of the information fusion process.

If we receive $Q_{SE_{j_i}}^{\#}$ at some node SE_j and there is a node DC_i without overriding assessment we combine all commonalities for DC_k using Eq. (19) with substitutions $A_i \to SE_j$ and $SE_j \to DC_k$, where $Q_{SE_j}^{\alpha_{SE_jDC_k}}(X)$ is substituted by $Q_{SE_j}^{\#}$ for those SE_j where overriding assessments are received. Notice the change in m_{SE_3} marked **SE 3** in Fig. 5 when the commander makes an overriding assessment with $m_{SE_j}^{\#}$.

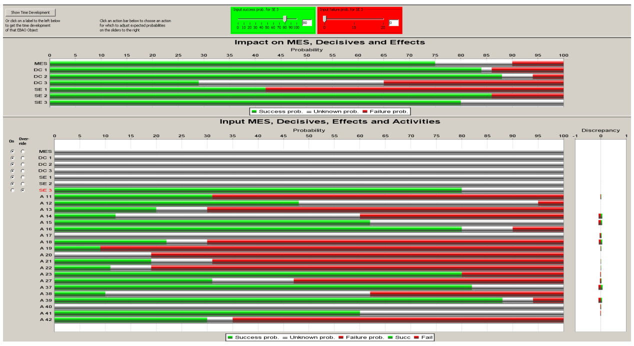

Fig. 5. Information fusion sweep (bottom-up) adding a commander's view.

4.2.2 *Information fission sweep*

This is similar to the case with combing assessments except that we use overriding assessments where they do exist. Whenever there is an overriding assessment, *e.g.*, at SE_j, we use it where we would otherwise have used the combined assessment.

Thus, we calculate $m_{A_i}^{*SE_j}(\mathrm{AdP})$ by substituting

$$Q^{\alpha_{A_i SE_j}}_{m_{SE_j} \oplus \{m_{A_i}^{\alpha_{ij}}\}_{i=1}^{n}}(\neg\mathrm{AdP}) \rightarrow Q^{\#}_{SE_j}(\neg\mathrm{AdP}) \qquad (32)$$

in Eq. (26), and calculate $m_{A_i}^{*SE_j}(\neg\mathrm{AdP})$ by substituting

$$Q^{\alpha_{A_i SE_j}}_{m_{SE_j} \oplus \{m_{A_i}^{\alpha_{ij}}\}_{i=1}^{n}}(\mathrm{AdP}) \rightarrow Q^{\#}_{SE_j}(\mathrm{AdP}) \qquad (33)$$

in Eq. (27).

Finally, we have

$$m_{A_i}^{*SE_j}(\Theta) = 1 - m_{A_i}^{*SE_j}(\mathrm{AdP}) - m_{A_i}^{*SE_j}(\neg\mathrm{AdP}). \qquad (34)$$

Again, it is possible that $m_{A_i}^{*}$ is a pseudo belief function in which case we may manage it in the same way as above.

5 Assessment discrepancy

We can measure the discrepancy between $m_{A_i}(X)$ and $m_{A_i}^{*SE_j}(X)$ for all A_i given SE_j and X, and similarly for all SE_i given DC_j and X, and all DC_i and X, where $X \in \{\mathrm{AdP}, \neg\mathrm{AdP}\}$.

However, we focus our attention on the maximum discrepancy (MD) for each A_i and SE_j within each family of belief functions, and the discrepancy for each DC_k.

We observe,

$$\begin{aligned} MD_{A_i}(X) &= max_j \left| m_{A_i}^{*SE_j}(X) - m_{A_i}(X) \right|, \quad \forall i \\ MD_{SE_i}(X) &= max_j \left| m_{SE_i}^{*DC_j}(X) - m_{SE_i}(X) \right|, \quad \forall i \\ MD_{DC_i}(X) &= \left| m_{DC_i}^{*MES}(X) - m_{DC_i}(X) \right|, \quad \forall i \end{aligned} \qquad (35)$$

where $X \in \{\mathrm{AdP}, \neg\mathrm{AdP}\}$, see Fig. 6.

A high discrepancy can be a sign of assessment error, or that some activity or effect does not fulfill its full potential.

6 A multi-level subjective EBA algorithm

With these calculations we have all pieces of a multi-level subjective EBA algorithm (Algorithm 1).

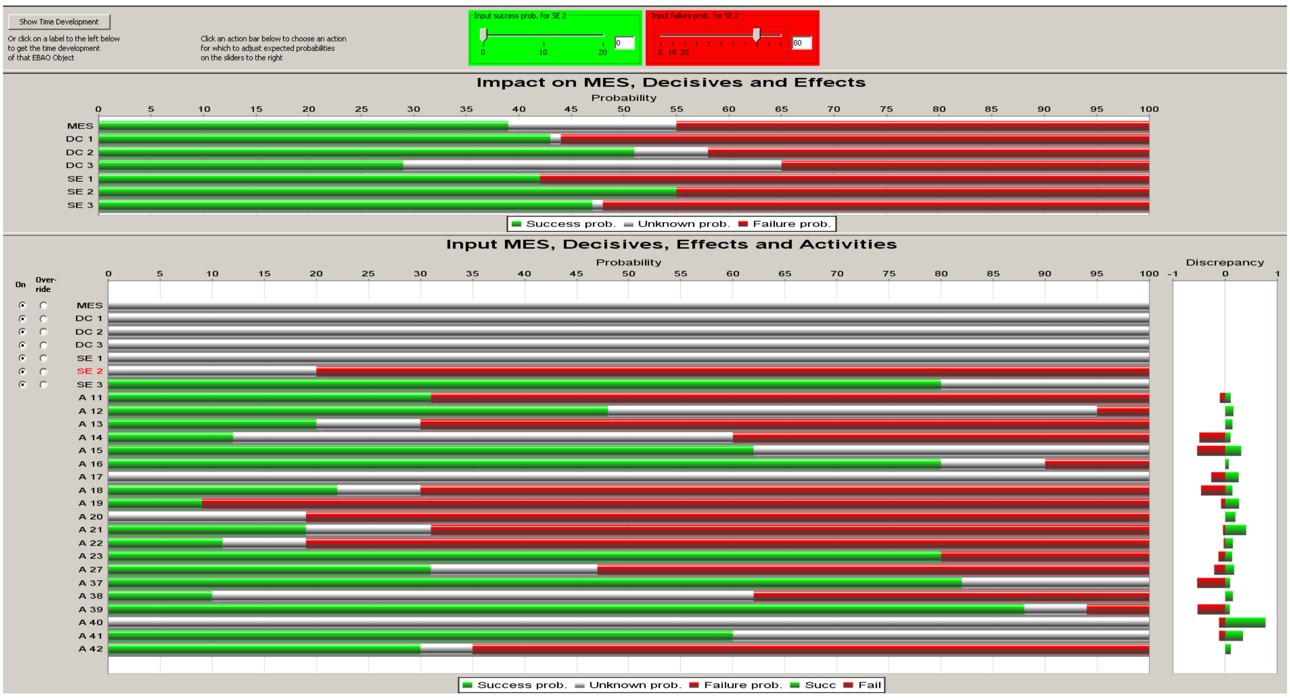

Fig. 6. The discrepancy to the right show the maximum additional assessment needed at the activity level to reach the effects-level assessments without the commander's view.

Algorithm 1: Multi-level Subjective EBA

Information fusion sweep (*bottom-up*),
Secs. 4.1.1 and 4.1.2:

- For all SE_i calculate:

  $m_{SE_i}(\text{AdP})$, $m_{SE_i}(\neg\text{AdP})$, $m_{SE_i}(\Theta)$.

- For all DC_i calculate:

  $m_{DC_i}(\text{AdP})$, $m_{DC_i}(\neg\text{AdP})$, $m_{DC_i}(\Theta)$.

- Calculate:

  $m_{MES}(\text{AdP})$, $m_{MES}(\neg\text{AdP})$, $m_{MES}(\Theta)$.

Information fission sweep (*top-down*),
Secs. 4.2.1 and 4.2.2:

- For all DC_i calculate:

  $m^{*\,MES}_{DC_i}(\text{AdP})$, $m^{*\,MES}_{DC_i}(\neg\text{AdP})$, $m^{*\,MES}_{DC_i}(\Theta)$.

- For all SE_i and DC_j calculate:

  $m^{*\,DC_j}_{SE_i}(\text{AdP})$, $m^{*\,DC_j}_{SE_i}(\neg\text{AdP})$, $m^{*\,DC_j}_{SE_i}(\Theta)$.

- For all A_i and SE_j calculate:

  $m^{*\,SE_j}_{A_i}(\text{AdP})$, $m^{*\,SE_j}_{A_i}(\neg\text{AdP})$, $m^{*\,SE_j}_{A_i}(\Theta)$.

Discrepancies, Sec. 5:

- For all A_i calculate:

  $MD_{A_i}(\text{AdP})$ and $MD_{A_i}(\neg\text{AdP})$.

- For all SE_i calculate:

  $MD_{SE_i}(\text{AdP})$ and $MD_{SE_i}(\neg\text{AdP})$.

- For all DC_i calculate:

  $MD_{DC_i}(\text{AdP})$ and $MD_{DC_i}(\neg\text{AdP})$.

- Return all calculated values.

7 Conclusions

We have developed a multi-level subjective Effects-Based Assessment method for making subjective assessment of plans and plan elements within the Effects-Based Approach to Operations.

We have shown that such subjective assessments can be performed of all supporting effects, decisive conditions and the military end state by taking human subjective assessments regarding all effect and activity levels as input, and combining and extending those assessments to all other plan elements using a cross impact matrix.

When we receive multiple-level assessments we can find the assessments necessary at lower levels needed to directly achieve the combined higher level assessment without the input received at the higher level itself. Comparing the discrepancy between the assessment we have, and the assessment we need tell us if some activities and effects are not matching up to what is needed to achieve success.

References

[1] Johan Schubert, Subjective effects-based assessment, in *Proceedings of the Eleventh International Conference on Information Fusion,* Cologne, Germany, 30 June–3 July, 2008, pp. 987–994.

[2] Johan Schubert, Analysis and assessment of effects-based plans, in *Proceedings of the NATO Symposium on Analytical Support to Defence Transformation,* Sofia, Bulgaria, 26–28 April 2010, paper 33, pp. 1–18.

[3] Arthur P. Dempster, Upper and lower probabilities induced by a multivalued mapping, *The Annals of Mathematical Statistics,* Vol. 38, No. 2, 325–339, April 1967.

[4] Arthur P. Dempster, A generalization of Bayesian inference, *Journal of the Royal Statistical Society Series B,* Vol. 30, No. 2, 205–247, 1968.

[5] Arthur P. Dempster, The Dempster-Shafer calculus for statisticians, *International Journal of Approximate Reasoning,* Vol. 48, No. 2, pp. 365–377, June 2008.

[6] Glenn Shafer, *A Mathematical Theory of Evidence*, Princeton University Press, Princeton, NJ, 1976.

[7] Glenn Shafer, Perspectives on the theory and practice of belief functions, *International Journal of Approximate Reasoning,* Vol. 4, No. 5/6, pp. 323–362, September–November 1990.

[8] Philippe Smets and Robert Kennes, The transferable belief model, *Artificial Intelligence,* Vol. 66, No. 2. pp. 191–234, March 1994.

[9] Philippe Smets, Practical uses of belief functions, in *Proceedings of the Fifteenth Conference on Uncertainty in Artificial Intelligence,* Stockholm, Sweden, 30 July–1 August 1999, pp. 612–621.

[10] Jerome C. Glenn and Theodore J. Gordon, Futures Research Methodology – Version 2.0. American Council for the United Nations University, Washington, DC, USA, 2003.

[11] Theodore J. Gordon and H. Hayward, Initial experiments with the cross-impact matrix method of forecasting, *Futures,* Vol. 1, No. 2, pp. 100–116, 1968.

[12] Edward A. Smith, *Complexity, networking, and effects-based approaches to operations,* U.S. Department of Defense CCRP, Washington, DC, 2006.

[13] Effects-based approach to multinational operations, Concept of operations (CONOPS) with implementation procedures. Version 1.0, United States Joint Forces Command, Suffolk, VA, USA, 31 July 2006.

[14] Johan Schubert, Mattias Wallén, and Johan Walter, Morphological Refinement of Effect Based Planning, in *Stockholm Contributions in Military Technology 2007,* Swedish National Defence College, Stockholm, Sweden, 2008, pp. 207–220.

[15] Incorporating and extending the UK military effects-based approach. Joint Doctrine Note (JDN 7/06), Development, Concepts and Doctrine Centre (DCDC), Ministry of Defence, Shrivenham, Swindon, UK, 2006.

[16] Philippe Smets, The canonical decomposition of a weighted belief, in *Proceedings of the 14th International Joint Conference on Artificial Intelligence,* Montréal, Canada, 20–25 August 1995, Vol. 2, pp. 1896–1901.