Skewed Distribution Analysis in Simulation-Based Operation Planning

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ABSTRACT

When providing decision support to commanders in military operation planning, one important task is to present decision makers with a quick and simple overview of which factors are important for success and within which parameter ranges success is achieved. We develop a methodology that simulates 10 000 different instances with uniform distributions over all input parameters, select the 1000 best simulations where blue forces achieve success according to the measures of effectiveness, and observe which parameters have skewed distributions within the smaller set of 1000. These are the important parameters, and the high frequency ranges of these parameters are the value ranges corresponding to blue force success.

1.0 INTRODUCTION

In this paper, we develop a *Commander's Overview* approach to decision making, providing a commander a quick and simple overview of the consequences of different decisions in military operation planning. The approach analyses large amounts of output data from simulations in a multiple-criteria decision support system focused on answering questions regarding force configuration, outcomes, etc. We assume a data farming [1] simulation experiment with multiple evaluation criteria has already taken place. Before addressing the commander's decision support, we must perform the multiple-criteria analysis and evaluate the entire data set using Measures of Effectiveness (MOEs) describing conditions for blue force success.

To provide decision support for a commander in operation planning, we perform a sequence of process steps. We first conduct scenario development, modeling, and simulation, followed by data analysis and decision support. The final step of decision support goes a step beyond what is traditionally performed within data farming. We think of this as data farming's decision support mode. In the operation-planning problem that we study, we have up to ten different MOEs. This gives us a multiple-criteria decision-making (MCDM) problem, which is managed by preference analysis of the MOEs, followed by a Monte Carlo weight assignment process. With this approach, we can avoid the difficult problem of weight assignment by human analysts and decision makers. With these two processes completed, we can focus on decision support. The decision support process is subdivided into three sub-processes: the *Analyst View* process, which is similar to the traditional statistical analysis typically performed in data farming; the *Commander's Overview* process, which is focused on the big picture of how to win in military combat; and the *Commander's Specific Questions* process, focusing on more specific questions of when we will win in different specific situations. In this paper, we focus on the *Commander's Overview* process. The other two processes are discussed in other papers.

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The fundamental idea of the *Commander's Overview* process is to select a subset of all simulation runs where blue forces achieve success as measured by the MOEs and observe any possible *skewedness* in the frequency distribution over the parameter ranges of all input parameters to the simulations in the remaining simulation subset, assuming a uniform distribution in the entire initial data set. In an experiment, we select the 1000 best simulation instances from the 10 000 instances simulated. Parameters with skewed distributions are considered important because outcomes vary over different ranges for these parameters, and we want to find what these parameters are and within what value ranges the parameters have a high frequency of being in the 1000-large set of simulations with preferable outcomes. To find the parameters with highly skewed distributions, we measure the entropy of the normalized frequency distribution of the parameters.

In Sec. 2, we present an overview of the data farming approach used to perform simulations. The ground warfare scenario used in this study is briefly described in Sec. 3. Then, in Sec. 4, we first describe three different processes of decision support to put the work of this paper into the context of previous work and then continue by developing the *Commander's Overview* approach, which focuses on the *Skewed Distribution Analysis*. Finally, some general conclusions are presented (Sec. 5).

2.0 DATA FARMING

Data farming [1] is a process aimed at maximizing the information available from a large set of data. The focus is on trying to produce a sufficiently complete landscape of potential outcomes rather than on identifying an individual response. In addition to identifying significant effects and relationships between input parameters, attention is also focused on detecting possible anomalies and including them in the decisions.

Data farming aims to provide insights into problem formulations and is an iterative process consisting of a *loop of loops*, as shown in Figure 1. In this paper, our focus is on data analysis and data visualization within operation planning.

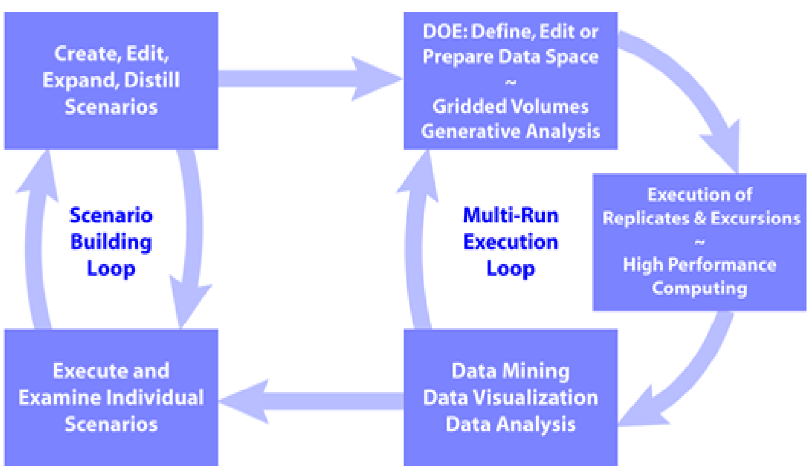

Figure 1: Scenario development and experimentation loop [1].

There is perhaps no optimal decision possible in a system where there are opponents acting in their own interests, but the notion is that more informed decisions can be made because the decision maker is allowed to understand the landscape of possibilities.

Based on the characteristics of the problem to be solved, there is a need for the modeling of nonlinearities, abstractions, and influence between the various parts of the problem in a functional manner. It is the combination of simple, efficient, and abstract models, as well as high-performance computing together with effective experiment design, that enables quick exploration of a solution space. Simple models make it easier to manage a large number of simulation runs, which enables exploration of a large parameter and value space and allows for the investigation of the solution space. The result is a landscape of outputs, which can then be used to analyze trends, detect anomalies, and generate insights regarding multiple parameter dimensions. In addition to identifying the general characteristics, the analysis also strives to provide understanding of the spread and central tendencies, as well as to elucidate internal parameter relationships and thresholds.

The core of data farming is based on a rich and diverse array of different simulation runs that are carried out on computers to check different assumptions, gain new insights into relevant relationships, and obtain more robust statements on opportunities and risks in specific mission situations. This is achieved by systematically varying the different parameter values for the input parameters that are assumed to be crucial to measures of effectiveness.

As mentioned, our focus is on data analysis and visualization and how we can use the data farming approach for decision-making generally and for decision support to a commander in operation planning specifically.

3.0 SCENARIO

Because we plan to employ the data farming method, we need to be able to create different versions of the scenario to act as input to our simulation runs. Hence, when modeling the scenario, we need a set of general variables that can be used for all versions of the scenario and a set of scenario-specific variables that depend on the current scenario terrain, units, tasks, and situations requiring tactical decisions.

To model the scenario, it is important to broaden the space of possible decisions that the simulated unit leaders can make and the actions that the simulated units can take. This is to avoid missing any possible and interesting situations when running simulations using a data farming approach.

Typically, a scenario in the context of operation planning is a description of the roles of different actors and their activities extended over a long period of time and over large areas. We initially focus on a smaller part of a scenario where only a few actors are active. This is called a vignette, which consists of a number of events together with actors who perform some specific activities, such as moving forward, reconnaissance, opening fire, etc.

The initial vignette includes a limited ground combat situation. Hence, the vignette used in our work unfolds in the context of defense against an armored attack, where an attack has been going on for a number of days before the start of the vignette. In this situation, red forces have air-dropped a parachute battalion at an airport and are in battle with a blue-force mechanized battalion reinforced with a tank company.

A new air-drop of a second red-force parachute battalion occurs in the areas around the village of Gimo (see Figure 2).

The task of this unit is to move south towards the airport to support the first red parachute battalion in ensuring that the airport stays open for the landing of transport aircraft carrying new military units. At the same time, a second blue mechanized battalion has regrouped and is positioned in the northern part of Uppsala. The task of this battalion is to prevent the red forces from reaching the airport further south.

Figure 2: Scenario map.

This part of the scenario is the focus of our simulation. The organization of the forces (Order of Battle − ORBAT) is as follows: a mechanized battalion on the blue side and a parachute battalion on the red side. In this example, these units are organized as described in Figure 3 and Figure 4. The units are simplified for the purpose of clarity.

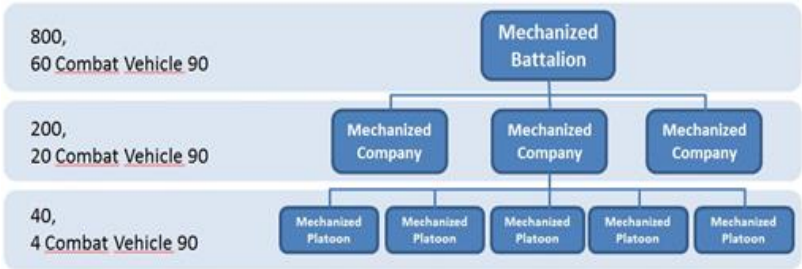

Figure 3: Blue order of battle. Left column are total man power and number of vehicles per unit.

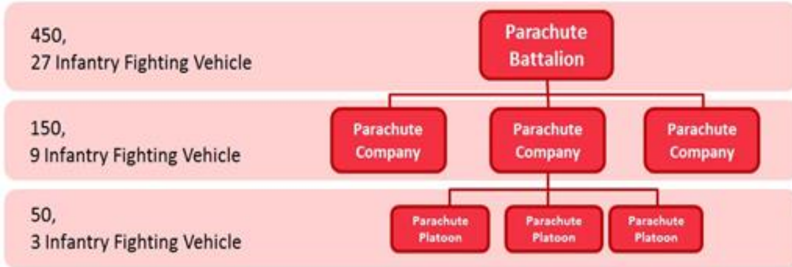

Figure 4: Red order of battle. Left column are total man power and number of vehicles per unit.

4.0 DECISION SUPPORT FOR THE COMMANDER

4.1 Decision Support Process Overview

The decision support processes are focused on identifying the most effective plan instances as evaluated by all measures of effectiveness (MOEs). When there are several MOEs, we are faced with an MCDM problem when assessing which plan instances are most preferred.

To find the best simulations, we must rank all simulation runs based on their MOEs. Thus, we need to assign weights to these measures. In some special situations, there might be sufficiently accurate knowledge of which parameter inputs and simulation outputs are most important to optimize to obtain a preferred outcome. When this is the case, it may be possible to directly assign weights to the different MOEs. However, this is usually not the case and is often a difficult problem for decision makers. As an alternative approach, we may let them express preferences on the relative importance between different MOEs or between any two disjoint subsets of MOEs.

In [2], we developed a preference-ranking approach as an extension to Utkin's [3] preference assignment method that is focused on finding the preferred order of importance of all MOEs from the preference assignments made by the decision makers. This extension uses interpolation in belief-plausibility intervals [4] regarding the obtained degree of preference of all different MOEs and delivers a complete ranking of all MOEs. The method accepts any number of preference expressions regarding the MOEs from any number of decision makers, for example, an expression such as *"MOE_i is more important than MOE_j"*; $MOE_i \succcurlyeq MOE_j$ or an expression regarding two different subsets of MOEs such as *"MOE_i and MOE_j are more important than MOE_k and MOE_l"*; $\{MOE_i, MOE_j\} \succcurlyeq \{MOE_k, MOE_l\}$.

Each MOE is ranked by

$$\frac{1}{2}\left[Bel_{\{i\}\Theta}(\{MOE_i\} \succcurlyeq \Theta) + Pls_{\{i\}\Theta}(\{MOE_i\} \succcurlyeq \Theta)\right] \tag{1}$$

where *Bel* and *Pls* are the belief and plausibility, respectively, and $\Theta = \{MOE_i\}$; see [2] for details.

The preference-ranking approach has been further extended to allow partial preference ranking of MOEs [5] using belief function theory [6–8]. Partial rankings of MOEs have higher belief-plausibility but less information value. It may be beneficial when the belief-plausibility is too low for the best complete rankings of MOEs. However, we use the complete ranking method [2] in this paper.

Using the preference order for all MOEs, we adopt a Monte Carlo approach to assign weights for these MOEs. We randomly assign weights that abide by the preferred order of the MOEs, i.e., the most preferred measure will be weighted higher than the second most preferred measure, etc. Using a Monte Carlo approach, we provide alternative weight assignments for all measures, yielding alternative rankings of all simulations. The simulations with the highest average ranking are the most preferred.

With this methodology, we obtain a ranking of all plans and can then analyze the best plans to learn which combination of parameter ranges leads to success. A process overview is provided in Figure 5.

Figure 5: Process overview.

The last process step in Figure 5, *Decision Support*, can be split into three sub-processes (Figure 6). They are:

- The *Analyst View* [9], where all data are statistically analyzed by an analyst who may prepare specific questions to be answered,
- The *Commander's Overview*, where the best simulations leading to blue success are analyzed to provide information regarding the number of input parameters needed to explain the result, which parameters these are, and what values these parameters should assume in order for blue to achieve success, and
- The *Commander's Specific Questions* [10], where subsets of simulations are analyzed by looking at a subset of parameter values for some red parameters or for some blue parameters.

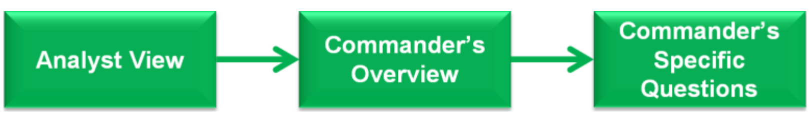

Figure 6: Three components of decision support.

The focus in this paper is on the *Commander's Overview*. The approaches taken in the *Analyst View* are developed in [9], and the approaches taken in the *Commander's Specific Questions* are developed in [10].

When analyzing the entire data set, we vary all input parameters independently, both concerning the red side and the blue side. Such an analysis constitutes an unrestricted data exploration and is usually a good first step to obtain an overview of the entire data set. This is what was performed in the *Analyst View* [9].

However, when we prefer an overview for the commander, we focus on the most important parameters leading to success. In this analysis, we want to find the important parameters: how many are they, and what are they? Both of these questions should be analyzed quantitatively by ranking alternative options by numeric evaluation. That is,

- We want to know what the preferred number of parameters is, what the second most preferred number of parameters is, etc. We also want to see a numeric evaluation of these alternative numbers of parameters, giving us an opportunity to compare the different alternatives, and
- For each alternative number of parameters of interest, we want to know what the most preferred group of parameters is, and what the second most preferred group of parameters is, etc. Again, we want to see numeric evaluations for the comparison of different groups of parameters.

Given that we have identified a few possibilities for the number of necessary parameters and what they might be, we are interested in finding within what ranges of these parameters we have blue success as defined by the MOEs.

This is the idea of the *Commander's Overview* sub-process: How many parameters do we need to achieve blue force success? What are these parameters? What are the value ranges for blue force success for these parameters?

4.2 Commander's Overview

4.2.1 Background of Skewed Distribution Analysis

Traditionally, data analysis within data framing has found important parameters and their value ranges by establishing a target function and analyzing this function. A target function can be a function of one MOE or a function of several, possibly weighted, MOEs. The target function specified is analyzed using a regression tree. This analysis yields a tree with alternative sequences of parameters at different branches of the tree with a two-way split of the parameter value range at each tree node. By stepping through the tree and deciding on the preferred value range, a sequence of the importance of parameters is found.

In the NATO study *Data Farming in Support of NATO* [10], we performed a case study on force protection. In this study, we defined a target function as a function of blue losses. However, it turned out that it was relatively easy in the simulation of the model to minimize the target function and achieve a global minimum of zero blue losses. This was achieved in 45% of all simulations. This caused the standard regression tree analysis to fail. As an alternative approach, the first author of this paper suggested a *Skewed Distribution Analysis* approach where we observe the frequency distributions of all discrete input parameters when studying a subset of simulations that achieved the global minimum. Continuous parameters were discretized. It was suggested that the importance of any parameter is equal to its distribution skewedness as measured by the Shannon entropy [12] of the normalized frequency of its values. This approach was not developed mathematically at the time and was used only visually within the NATO study.

4.2.2 Conceptual Idea of Skewed Distribution Analysis

The idea behind *Skewed Distribution Analysis* is that, if a parameter is important, there must be a decisive decision regarding that parameter that differentiates between success and failure. This implies that, if we partition the set of all simulation runs into two subsets, one for success and one for failure, we should observe this parameter to take different values in the two subsets. Specifically, the frequency of values within the subset should be highly different. Thus, within each subset, the distribution of values should be highly skewed. The analysis can also be performed for several parameters where we measure the combined parameter skewedness by their joint entropy.

4.2.3 Entropy Approach to Skewed Distributions

We can estimate the deviation of a particular distribution of interest from a corresponding uniform distribution over the same parameter by comparing its entropy with the uniform distribution. In this approach, we study the entropy of discrete distributions for parameters whose possible values are enumerated with integers. This is performed for the best 10% of all simulations as measured by MOEs [10] (e.g., on the best 1000 simulations of all 10 000 simulations) to quantify the skewedness of the input parameters. For example,

- When a parameter can take up to a maximum of 20 different values, we chose these values as the possible states for that parameter, and

- When the number of possible values of a parameter exceeds 20, we partition the parameter range into 20 different states.

The entropy H of a discrete probability distribution P of a parameter X_i that can take N_i discrete states is defined as

$$H_i = -\sum_{a=1}^{N_i} \mathrm{P}(x_{ia}) \log_2 \mathrm{P}(x_{ia}), \tag{2}$$

where $\mathrm{P}(x_{ia})$ is the probability for state a of parameter *i*. A uniform probability distribution with $x_{ia} = 1/N_i, 1 \leq a \leq N_i$ yields the highest entropy, for which (2) reduces to

$$H_{i,max} = -\sum_{j=1}^{N_i} \frac{1}{N_i} \log_2 \left(\frac{1}{N_i}\right) = \log_2 N_i. \tag{3}$$

Because the lowest entropy is reached when a parameter occupies a single state, e.g., $x_{i1} = 1, x_{i1}, \ldots, x_{iN_i} = 0$ $\Rightarrow H_{i,min} = 0$, we have $0 \leq H_i \leq \log_2 N_i$. Within this interval, we can regard the analyzed distribution as being farther from uniform the closer H is to zero. This does not necessarily mean higher skewedness because skewedness in some sense implies x_a monotonically changing with increases in *a*. However, because the sum of (2) is invariant with respect to the summation order of *a*, we can obtain the same entropy for a *spiky* distribution, which looks like noise. If the analyzed parameter is ordinal, i.e., with an inherent order (such as the number of something, a distance, a weight, etc.), this can mean that the dependence on that parameter is weak because noise seems to dominate. In cases of nominal (categorical) parameters, the order of the possible states has no meaning. One example is a distribution for which 10 different transportation routes are the most advantageous for an actor to follow in a large number of simulations.

When measuring skewedness of a distribution, we prefer all input parameter values to be sampled from uniform distributions. However, because imperfections in the sampling of input parameters for each simulation case can occur, we correct for these by dividing the frequencies of input parameter values for the best 1000 simulations with the corresponding frequencies for the full set of 10 000 simulations.

As a final adjustment to obtain a better understanding of skewnesses, we *normalize* all entropies by dividing them with $H_{i,max}$ for each distribution *i*. We obtain normalized entropy via

$$\overline{H}_i = \frac{H_i}{H_{i,max}}. \tag{4}$$

The entropy calculation can be extended to the *normalized joint entropy* of two or several parameter distributions. The rationale for this is to study which combinations (*tuples*) of parameters are most important to pay attention to when trying to find *good* simulations. The number of possible states will, in those cases, be the product of the number of states for the singular distributions. For a pair (2-tuple) of parameters $\{x_i, x_j\}$, a simulation is in some joint state $\{x_{ia}, x_{jb}\}$ where $1 \leq a \leq N_i$, $1 \leq b \leq N_j$.

Analogously, for 2-tuples, there are $N_i \cdot N_j$ joint states to sum over in the calculation of the joint entropy H_{ij}. We have,

$$H_{ij} = -\sum_{a=1}^{N_i}\sum_{b=1}^{N_j} \mathrm{P}(x_{ia}, x_{jb}) \log_2 \mathrm{P}(x_{ia}, x_{jb}). \quad (5)$$

We obtain the maximum joint entropy $H_{ij,max} = \log_2(N_i \cdot N_j)$ and calculate the normalized joint entropy for 2-tuples as

$$\overline{H}_{ij} = \frac{H_{ij}}{H_{ij,max}}. \quad (6)$$

The 2-tuples approach can easily be extended for 3-tuples, 4-tuples, etc.

4.2.4 How to Find Important Parameters and Their Ranges

The above simulation and analysis form a basis for identifying the variables, and the associated values, that most decisively yield a desirable operational outcome.

In the previous subsection, we calculated the normalized joint entropy for a selection of parameters. Low normalized joint entropy means a highly skewed distribution and suggests that the operational outcome is highly sensitive to the value selection for that parameter (or set of parameters). The minimum entropy value is zero when only one value leads to success. Conversely, high normalized joint entropy indicates that the simulation result is more or less independent on the value of that parameter and that the parameter in question may be ignored in further analysis.

A first step to isolate the most decisive parameters and values is to look at the normalized joint entropies of all combinations of parameters. In the previous section, we described the computation of the normalized entropy of single parameter distributions (1-tuples) (4) and pairs of parameter distributions (2-tuples) (6). Generally, we call such combinations *k*-tuples.

In Figure 7, we see the minimum normalized joint entropy over all *k*-tuples from one to 16 parameters.

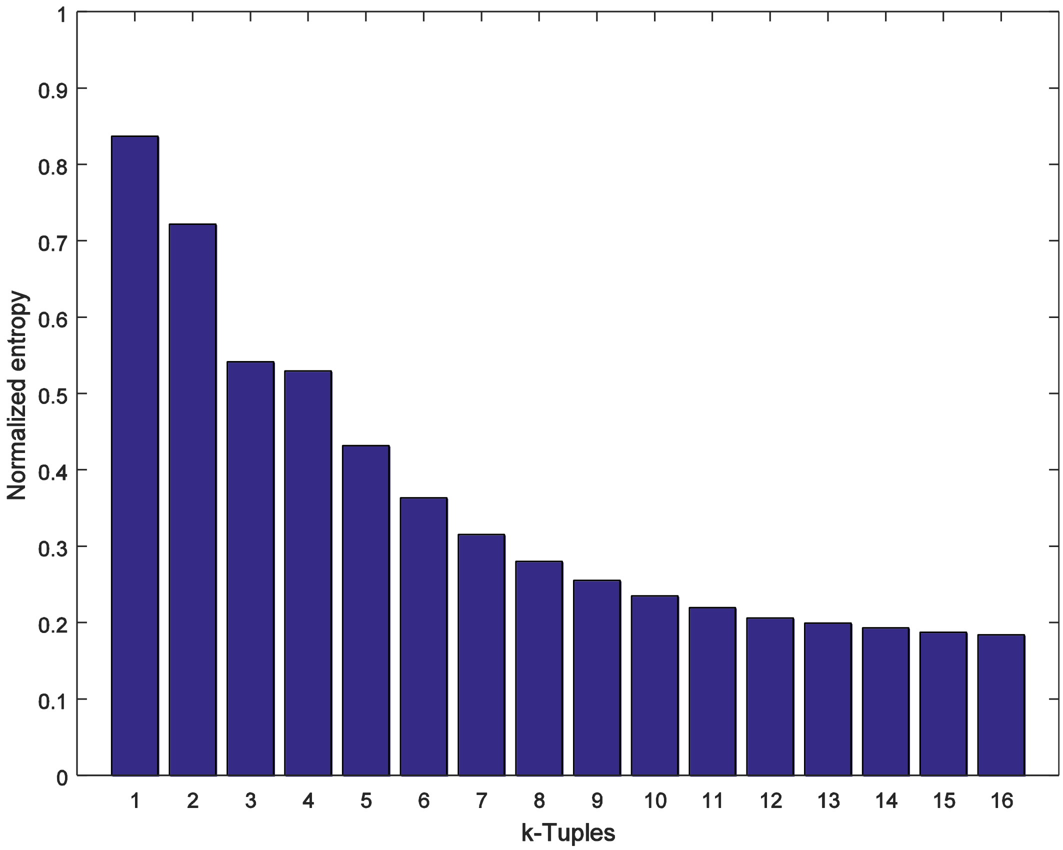

Figure 7: The minimum joint entropies for all *k*-tuples from *k* = 1 to *k* = 16.

As seen, the minimum normalized joint entropy apparently decreases monotonically as the number of parameters increases. For example, $\min_{ij} \overline{H}_{ij} \leq \min_{i} \overline{H}_{i}$, etc. This is because $H_{ij,max}$ (the denominator) increases faster than H_{ij} (the numerator) in equation (6) (note that, while the inequalities $\max(H_i, H_j) \leq H_{ij} \leq H_i + H_j$ hold for unnormalized entropies, this is not the case for normalized entropies (4, 6)). However, although it appears advantageous to consider more and more parameters, this may be costly in general. Hence, at some point (e.g., for some threshold, or when the normalized entropy decrease is small enough), it is sensible to stop considering more parameters. From Figure 7, it appears that considering three parameters may be worthwhile, but adding a fourth parameter does not add much.

Once the number of parameters k to consider has been established, we can begin to consider which are the preferred parameters and study their optimal value ranges. In Figure 8, we present parameters and parameter value ranges yielding the lowest entropy.

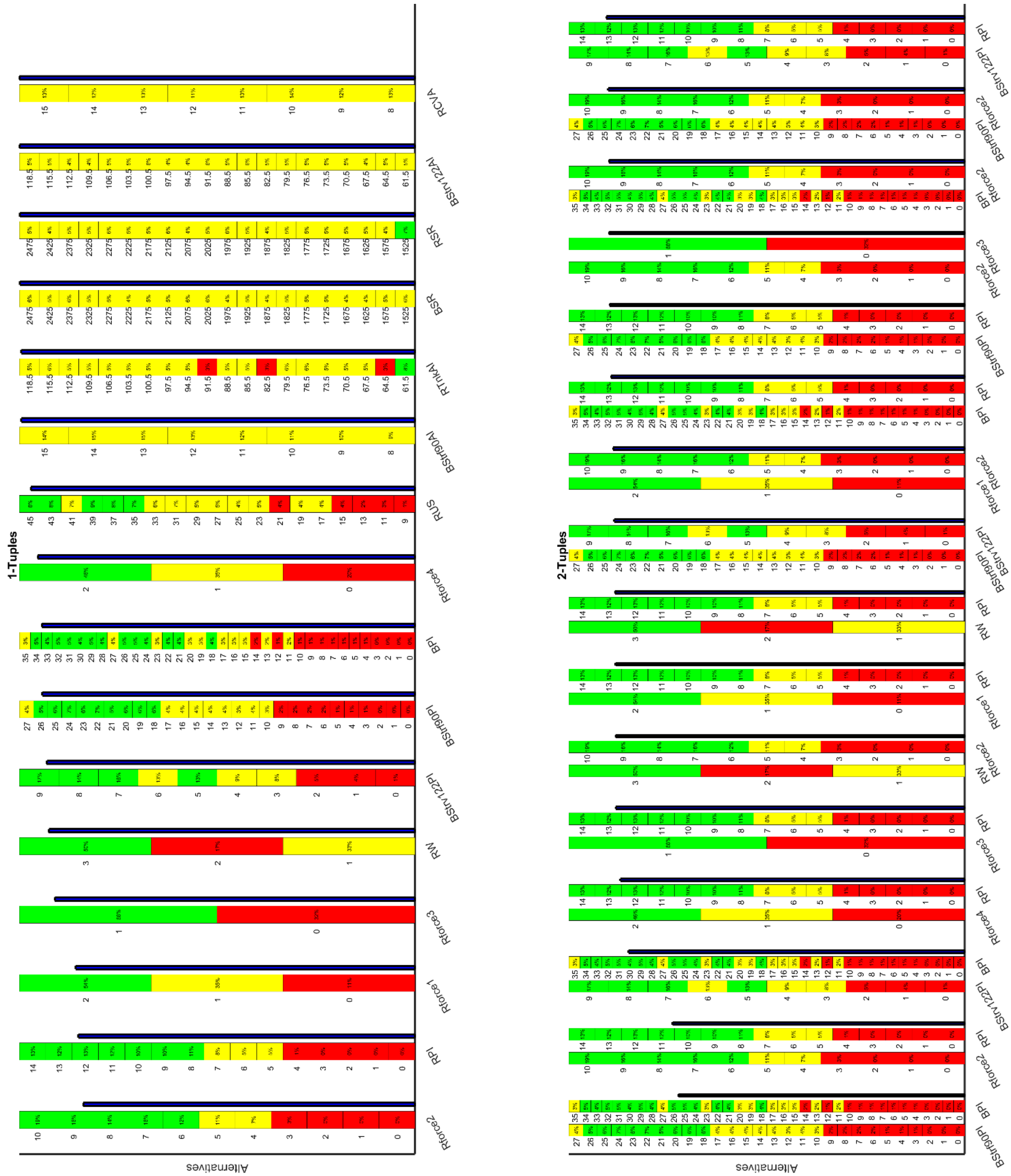

Figure 8: Skewed Distribution Analysis of 16 input parameters (1-tuples) and the 16 best 2-tuples (rotated 90º counter clockwise).

On the first row, we observe distributions for single parameters (1-tuples) with the lowest normalized joint entropy depicted as a bar consisting of a stack of cells. Each cell represents a parameter value or a parameter interval. The parameter value, or the lower interval edge, is shown to the left of the cell. The value inside the cell is the frequency of that particular value in the skewed distribution. The frequency value is emphasized by the color of the cell: red for low frequencies, green for high, and yellow for frequencies in between. To the right of each parameter stack is a thin blue bar, which represents the normalized joint entropy value (4). Note that the maximum height of the blue normalized joint entropy bar (H_{max}) is the same for all *k*-tuples on each row for easy comparison.

On the second row, we show pairs of parameters (2-tuples) with the lowest normalized joint entropy value in increasing order. Note that, in the visualization (in Figure 8, row 2), we present several pairs of bars with two marginal distributions rather than single bars with joint distributions. Also note that *k*-tuples with sizes larger than two can be presented (although they are not shown in the figure). The total number of bar groups for each *k*-tuple size is the binomial coefficient

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!}, \tag{7}$$

where, in our case, the number of parameters $n = 16$. Already at $k = 2$, the number of 2-tuples bars is 120 (only the first 16 are shown).

Given the preferred number of parameters, we can study various parameter tuples on each row. Those on the left have the lowest normalized joint entropy and tend to have skewed distributions with particular parameter ranges of importance. On the right of Figure 8, we have parameters that did not turn out to be decisive for the outcome of the simulations.

4.2.5 An Example of Skewed Distribution Analysis

The previous subsection outlines the general procedure for how to work with skewed distributions. Let us exemplify this by looking at actual parameters and values of Figure 7 and Figure 8.

We start by looking at the minimum normalized joint entropy for *k*-tuples in Figure 7. We find an initial sharp decrease in normalized joint entropy but only small improvements from tuples of size eight or so. In this case, the 1-tuple with the minimum normalized joint entropy is *Rforce2*; the 2-tuple with the minimum joint entropy consists of the parameters *BStrf90PI* and *BPI*, as shown in Figure 8. As the normalized joint entropy reflects how decisive a *k*-tuple is, we might want to select *k*-tuples with low normalized joint entropy but with as small a *k* as possible, as a large *k* means a more complex tuple to address and a longer time to generate it.

Looking at the 1-tuples in Figure 8, if those are the commander's choice, we see that the first three (*Rforce2*, *RPI* and *Rforce1*; row 1) have similar normalized joint entropies. They are therefore of roughly equal importance. If the result of the simulations should be understood from just one parameter, here are the options. In the case of *Rforce2*, we can see that a majority of the best simulation outcomes have values in the range of six to ten (colored green).

For 1-tuples, the first three consist of input parameters for the simulation regarding the red force. The 2-tuples in Figure 8 show that the most preferred parameters for 1-tuples may not necessarily be included in the preferred tuples when the tuple size is increased to two. For 2-tuples, the tuple with the minimum normalized joint entropy includes two parameters for the blue force.

The best 1-tuple and the best 2-tuple are two alternative ways of explaining how the MOEs can be optimized. Either *Rforce2* assumes an optimum value, or alternatively both *BStrf90PI* and *BPI* assumes optimum values simultaneously. All other 1-tuples and 2-tuples are also possible explanations for how to optimize the MOEs, but to a lesser degree as the have higher entropies. In general, a k-tuple with low k is preferred unless a more complicated way of optimizing the MOEs by an m-tuple ($k < m$) is more cost efficient.

5.0 CONCLUSIONS

We have presented a decision support methodology for a *Commander's Overview* that presents a big-picture overview of simulation results, describing which parameters are important for success and within what ranges these parameters must lie to achieve success in operation planning. We consider this methodology, together with the previously developed *Analyst View* [9] and *Commander's Specific Questions* [10], to be a first step towards taking the data farming methodology from its traditional analytical view and applying it in an operation-planning context and a decision-making mode.

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