Subjective Effects-Based Assessment

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Abstract - *In this paper we develop a subjective Effects-Based Assessment method. This method takes subjective assessments regarding the activities of a plan as inputs. From these assessments and a cross impact matrix that represents the impact between all elements of the plan we calculate assessments for all other plan elements. The method is based on belief functions and their combination under a new generalization of the discounting operation. The method is implemented in a Collaboration Synchronization Management Tool (CSMT).*

Keywords: Effects-Based Approach to Operations, Effects-Based Assessment, Subjective assessment, Dempster-Shafer theory, belief function, pseudo belief function, decomposition, inverse support function.

1 Introduction

In this paper we develop a subjective method for *Effects-Based Assessment* (EBA) based on belief functions [1−6] and a *cross impact matrix* (CIM) [7, 8]. This work extends our previous work [9] on analyzing plans developed within the *Effects-Based Planning* (EBP) [10] process.

A CIM can be used on the operational command level by the staff of a joint task force headquarter in an *Effects-Based Approach to Operations* [11] during planning, execution and assessment of an operation. The CIM consists of all *activities* (A), *supporting effects* (SE), *decisive conditions* (DC) and *military end state* (MES) of the plan. It is created by a broad working group which must assess how each activity impacts every other activity and supporting effect, how each supporting effect impacts every decisive condition (and possibly other supporting effects), and how every decisive condition impacts the military end state (and possibly other decisive conditions). In this paper we use British concepts [12].

The CIM can be used during assessment of the operation as it should contain the most current view of what impact all supporting effects have on the decisive conditions and what impact all decisive conditions have on the military end state.

Accepting human subjective assessments regarding the successful outcome of activities of the plan, we can use the impacts between plan elements as described by the CIM to calculate similar subjective assessments of all desired supporting effects, decisive conditions and the military end state. Using this methodology we get an early assessment of all plan elements during *Effects-Based Execution* (EBE) and may early on observe if activities and desired effects are developing according to plan. By observing the change over time of these subjective assessments of effects and conditions as assessments of activities are updated, we notice if trends are moving in the right direction as more activities are further executed.

In Sec. 2 we describe the construction of a CIM. In Sec. 3 we develop an algorithm for assessment of plan elements using a CIM, and show how this may be used for subjective assessment of all desired effects. In Sec. 4 we decompose [13−15] assessments into separate statements for and against the realization of the desired effects. Finally, in Sec. 5 conclusions are drawn.

2 The plan an the CIM

The cross impact matrix will initially be created during the planning process. It should be created by a working group containing key subject matter experts as required by the type of operation planned. Before the CIM is constructed, a plan must be formed according to EBP. The plan consists of a military end state, decisive conditions, supporting effects and activities, Fig. 1.

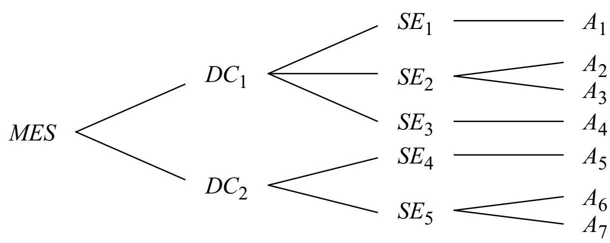

Fig. 1. Effects-based planning: *MES* = military end state, *DC* = decisive condition, *SE* = supporting effect, *A* = activity.

The working group will first need to enter all planned activities into the CIM, and it is important that all activities are well defined. They will then have to decide which positive or negative impact each activity will have on every other activity. It is important to note that even if activity A_1 has a positive impact on activity A_2, A_2 could have a negative impact on A_1. In the next step the working group

in *Proc. 11th Int. Conf. Information Fusion* (FUSION 2008),
pp. 987–994, Cologne, Germany, 30 June - 3 July 2008.

must decide what impact all activities have on the supporting effects, what impact all supporting effects have on the decisive conditions and what impact the decisive conditions have on the military end state.

In Fig. 2 we will list these elements, except the military end state, to the left of the CIM and list the elements, including the military end state, above the CIM. The CIM consists of values ranging from -9 to 9, where -9 denotes large negative influence, 0 means no influence and 9 denotes high positive influence. For example, an impact value of 8, i.e., “high positive influence”, might be assigned between the activity of “securing an area” and the activity of “transporting through that area”. How much element i influences element j is stored in the cell (i, j) in the CIM (for example activity A_2 influences activity A_4 in a positive way by a factor of 2, but A_4 influences A_2 in a negative way by a factor of -2). Its value is accessible through the function impact(i, j).

	Military End State	DC_1	DC_2	SE_1	SE_2	A_1	A_2	A_3	A_4
DC_1	5	0	6	0	0	0	0	0	0
DC_2	8	6	0	0	0	0	0	0	0
SE_1	0	5	2	0	0	0	0	0	0
SE_2	0	5	8	0	0	0	0	0	0
A_1	0	0	0	3	3	0	4	-2	-3
A_2	0	0	0	3	-2	3	0	0	2
A_3	0	0	0	3	6	0	8	0	0
A_4	0	0	0	4	-2	-7	-2	0	0

Fig. 2. The CIM contains a military end state, decisive conditions, supporting effects and activities (dark gray cells always contain zeros). The CIM is the impact(i, j) matrix.

At the initial stage of the construction of the CIM we include the basic elements of the plan meaning that all activities should be performed and all supporting effects and decisive conditions should be reached.

3 Assessment of plan elements

The CIM is a model of influence between elements of the plan. In assessment, our interest is on the impact between activities on the lowest level and supporting effects on the next level, and so forth. We receive subjective assessments regarding activities as user input. These are in the form of basic belief assignments (bbas) that express support for and against the success of that activity, encoded as AdP and ¬AdP, respectively.

3.1 Combining assessments

In this problem we have a simple frame of discernment

$$\Theta = \{\mathrm{AdP}, \neg\mathrm{AdP}\} \tag{1}$$

on each hierarchical level of the plan, where AdP means an *Adequate Plan*.

We have a set of n bbas each with three bodies of evidence, i.e., $\{\{(\mathrm{AdP}, m_i(\mathrm{AdP}))\,,\ (\neg\mathrm{AdP}, m_i(\neg\mathrm{AdP}))\,,\ (\Theta, 1 - m_i(\mathrm{AdP}) - m_i(\neg\mathrm{AdP}))\}\}_{i=1}^{n}$, where, e.g., $(\mathrm{AdP}, m_i(\mathrm{AdP}))$ is the first body of evidence of the ith bba giving support to AdP. Thus, for the ith bba we have,

$$m_i(A) = \begin{cases} m_i(\mathrm{AdP}), & A = \mathrm{AdP} \\ m_i(\neg\mathrm{AdP}), & A = \neg\mathrm{AdP} \\ 1 - m_i(\mathrm{AdP}) - m_i(\neg\mathrm{AdP}), & A = \Theta \end{cases} . \tag{2}$$

The CIM contains all information regarding the impact of each activity on all supporting effects. When the impact on a particular supporting effect SE_j is less than full we discount the bba m_i in relation to its degree of impact on SE_j

$$m_i^{\alpha_{ij}}(A) = \begin{cases} \alpha_{ij} m_i(\mathrm{AdP}), & A = \mathrm{AdP} \\ \alpha_{ij} m_i(\neg\mathrm{AdP}), & A = \neg\mathrm{AdP} \\ 1 - \alpha_{ij} m_i(\mathrm{AdP}) - \alpha_{ij} m_i(\neg\mathrm{AdP}), & A = \Theta \end{cases} . \tag{3}$$

For the sake of simplicity in combination of bbas, they are first transformed to commonalities using

$$Q(A) = \sum_{B \supseteq A} m(B) . \tag{4}$$

Transforming all bbas $m_i^{\alpha_{ij}}(B)$ to commonalities $Q_i^{\alpha_{ij}}(A)$ using Eq. (4), we have

$$Q_i^{\alpha_{ij}}(A) = \begin{cases} 1 - \alpha_{ij} m_i(\neg\mathrm{AdP}), & A = \mathrm{AdP} \\ 1 - \alpha_{ij} m_i(\mathrm{AdP}), & A = \neg\mathrm{AdP} \\ 1 - \alpha_{ij} m_i(\mathrm{AdP}) - \alpha_{ij} m_i(\neg\mathrm{AdP}), & A = \Theta \end{cases} . \tag{5}$$

Let us now combine all commonalities $Q_i^{\alpha_{ij}}$, using Dempster’s rule for commonalities. We get

$$Q^{\alpha_{ij}}_{\oplus\{m_i\}_{i=1}^{n}}(A) =$$

$$= \begin{cases} K \prod_i [1 - \alpha_{ij} m_i(\neg\mathrm{AdP})], & A = \mathrm{AdP} \\ K \prod_i [1 - \alpha_{ij} m_i(\mathrm{AdP})], & A = \neg\mathrm{AdP} \\ K \prod_i [1 - \alpha_{ij} m_i(\mathrm{AdP}) - \alpha_{ij} m_i(\neg\mathrm{AdP})], & A = \Theta \end{cases} \tag{6}$$

where K is a normalizing constant.

Commonalities can be transformed back to bbas using

$$m(A) = \sum_{B \supseteq A} (-1)^{|B - A|} Q(B) . \tag{7}$$

Transforming back from commonality to bba, we get

$$m^{\alpha_{ij}}_{\oplus\{m_i\}_{i=1}^{n}}(A) =$$

$$= \begin{cases} K\left\{\prod_i [1-\alpha_{ij}m_i(\neg\text{AdP})] \right. \\ \qquad \left. -\prod_i [1-\alpha_{ij}m_i(\text{AdP})-\alpha_{ij}m_i(\neg\text{AdP})]\right\}, & A = \text{AdP} \\ K\left\{\prod_i [1-\alpha_{ij}m_i(\text{AdP})] \right. \\ \qquad \left. -\prod_i [1-\alpha_{ij}m_i(\text{AdP})-\alpha_{ij}m_i(\neg\text{AdP})]\right\}, & A = \neg\text{AdP} \\ K\prod_i [1-\alpha_{ij}m_i(\text{AdP})-\alpha_{ij}m_i(\neg\text{AdP})], & A = \Theta \end{cases} \tag{8}$$

where

$$\begin{aligned} K^{-1} = {} & \prod_i [1-\alpha_{ij}m_i(\neg\text{AdP})] \\ & -\prod_i [1-\alpha_{ij}m_i(\text{AdP})-\alpha_{ij}m_i(\neg\text{AdP})] \\ & +\prod_i [1-\alpha_{ij}m_i(\text{AdP})] \\ & -\prod_i [1-\alpha_{ij}m_i(\text{AdP})-\alpha_{ij}m_i(\neg\text{AdP})] \\ & +\prod_i [1-\alpha_{ij}m_i(\text{AdP})-\alpha_{ij}m_i(\neg\text{AdP})]. \end{aligned} \tag{9}$$

Thus, Eq. (8) becomes the subjective assessment of SE_j as calculated using the subjective input assessments of all activities A_i that impact upon SE_j.

What is calculated for supporting effects from subjective assessment of activities can in a second phase be calculated for decisive conditions using the newly calculated assessments of supporting effects. In the same way we can calculate the subjective assessment of the military end state from the assessment of decisive conditions.

3.2 Combining assessments regarding plan elements using the CIM.

At the activities level we have a frame of discernment

$$\Theta_A = \{\text{AdP}, \neg\text{AdP}\}. \tag{10}$$

In order to map this onto the problem of combining assessments, Sec. 3.1, we must first generalize the discounting operation.

The discounting operation was introduced to handle the case when the source of some piece of evidence is lacking in credibility [3]. The credibility of the source, $0 < \alpha < 1$, also became the credibility of the piece of evidence. The situation was handled by discounting each supported proposition other than Θ with the credibility α and by adding the discounted mass to Θ;

$$m^{\%}(A) = \begin{cases} \alpha m(A), & A \neq \Theta \\ 1-\alpha+\alpha m(\Theta), & A = \Theta \end{cases}. \tag{11}$$

We generalize the discounting operation by allowing the credibility to take values in the interval $-1 \le \alpha < 1$.

Definition 1. Let $m: 2^{\Theta} \to [0, 1]$ be a bba where $-1 \le \alpha < 1$. Then

$$m^{\%}(A) = \begin{cases} \alpha m(A), & A \neq \Theta \\ 1-\alpha+\alpha m(\Theta), & A = \Theta \end{cases}. \tag{12}$$

is a generalized discounting of m where $m^{\%}(A)$ is an inverse simple support function (ISSF) whenever $\alpha < 0$.

Definition 2. An inverse simple support function on a frame of discernment Θ is a function $m: 2^{\Theta} \to (-\infty, \infty)$ characterized by a weight $w \in (1, \infty)$ and a focal element $A \subseteq \Theta$, such that $m(\Theta) = w$, $m(A) = 1 - w$ and $m(X) = 0$ when $X \notin \{A, \Theta\}$.

Let us recall the meaning of simple support functions (SSFs) and ISSFs, [13]: An SSF $m_1(\text{A}) \in [0, 1]$ represents a state of belief that "You have some reason to believe that the actual world is in A (and nothing more)". An ISSF $m_2(\text{A}) \in (-\infty, 0)$, on the other hand, represents a state of belief that "You have some reason *not* to believe that the actual world is in A".

Before combining the mass functions we discount them using the impact values of the CIM. This ensures that each activity influences the supporting effect to its proper degree.

For SE_j and A_i we have

$$m^{\alpha_{kj}}_{A_i}(A)$$

$$= \begin{cases} \alpha_{A_iSE_j} m_{A_i}(\text{AdP}), & A = \text{AdP} \\ \alpha_{A_iSE_j} m_{A_i}(\neg\text{AdP}), & A = \neg\text{AdP} \\ 1-\alpha_{A_iSE_j} m_{A_i}(\text{AdP})-\alpha_{A_iSE_j} m_{A_i}(\neg\text{AdP}), & A = \Theta \end{cases} \tag{13}$$

where the discounting factor is defined as

$$\alpha_{A_iSE_j} \triangleq \frac{\text{impact}(A_i, SE_j)}{10}. \tag{14}$$

This is a generalization of the discounting operator where discounting factors may assume values less than 0, i.e., $\alpha_{kj} = \{-0.9, -0.8, -0.7, \ldots, 0.9\}$.

We combine all bbas on the activities level and bring the result to the supporting effects level. At the supporting effects level we have a similar frame of discernment

$$\Theta_{SE} = \{\text{AdP}, \neg\text{AdP}\}. \tag{15}$$

Using Eq. (8), Eq. (13) and Eq. (14), we define

$$m_{SE_j}(\mathrm{AdP}) \triangleq m^{\alpha_{ji}}_{\oplus\{m_{A_i}\}_{i=1}^n}(\mathrm{AdP})$$
$$= K\Big\{\prod_i [1-\alpha_{A_iSE_j} m_{A_i}(\neg\mathrm{AdP})]$$
$$-\prod_i [1-\alpha_{A_iSE_j} m_{A_i}(\mathrm{AdP})-\alpha_{A_iSE_j} m_{A_i}(\neg\mathrm{AdP})]\Big\}$$
$$= K\Big\{\prod_i \Big[1-\frac{\mathrm{impact}(A_i, SE_j)}{10} m_{A_i}(\neg\mathrm{AdP})\Big] \tag{16}$$
$$-\prod_i \Big[1-\frac{\mathrm{impact}(A_i, SE_j)}{10} m_{A_i}(\mathrm{AdP})$$
$$-\frac{\mathrm{impact}(A_i, SE_j)}{10} m_{A_i}(\neg\mathrm{AdP})\Big]\Big\}$$

and

$$m_{SE_j}(\neg\mathrm{AdP}) \triangleq m^{\alpha_{ji}}_{\oplus\{m_{A_i}\}_{i=1}^n}(\neg\mathrm{AdP})$$
$$= K\Big\{\prod_i [1-\alpha_{A_iSE_j} m_{A_i}(\mathrm{AdP})]$$
$$-\prod_i [1-\alpha_{A_iSE_j} m_{A_i}(\mathrm{AdP})-\alpha_{A_iSE_j} m_{A_i}(\neg\mathrm{AdP})]\Big\}$$
$$= K\Big\{\prod_i \Big[1-\frac{\mathrm{impact}(A_i, SE_j)}{10} m_{A_i}(\mathrm{AdP})\Big] \tag{17}$$
$$-\prod_i \Big[1-\frac{\mathrm{impact}(A_i, SE_j)}{10} m_{A_i}(\mathrm{AdP})$$
$$-\frac{\mathrm{impact}(A_i, SE_j)}{10} m_{A_i}(\neg\mathrm{AdP})\Big]\Big\}$$

with

$$m_{SE_j}(\Theta) \triangleq m^{\alpha_{ji}}_{\oplus\{m_{A_i}\}_{i=1}^n}(\Theta)$$
$$= K\prod_i [1-\alpha_{A_iSE_j} m_{A_i}(\mathrm{AdP})-\alpha_{A_iSE_j} m_{A_i}(\neg\mathrm{AdP})]$$
$$= K\prod_i \Big[1-\frac{\mathrm{impact}(A_i, SE_j)}{10} m_{A_i}(\mathrm{AdP}) \tag{18}$$
$$-\frac{\mathrm{impact}(A_i, SE_j)}{10} m_{A_i}(\neg\mathrm{AdP})\Big]$$

For simplicity, we may manage the normalization by first calculating

$$m^*_{SE_j}(\mathrm{AdP}) = \frac{m_{SE_j}(\mathrm{AdP})}{K}$$
$$m^*_{SE_j}(\neg\mathrm{AdP}) = \frac{m_{SE_j}(\neg\mathrm{AdP})}{K} \tag{19}$$
$$m^*_{SE_j}(\Theta) = \frac{m_{SE_j}(\Theta)}{K}$$

using Eq. (16), Eq. (17), and Eq. (18), followed by calculating

$$m_{SE_j}(\mathrm{AdP}) = \frac{m^*_{SE_j}(\mathrm{AdP})}{m^*_{SE_j}(\mathrm{AdP})+m^*_{SE_j}(\neg\mathrm{AdP})+m^*_{SE_j}(\Theta)}$$
$$m_{SE_j}(\neg\mathrm{AdP}) = \frac{m^*_{SE_j}(\neg\mathrm{AdP})}{m^*_{SE_j}(\mathrm{AdP})+m^*_{SE_j}(\neg\mathrm{AdP})+m^*_{SE_j}(\Theta)} \tag{20}$$
$$m_{SE_j}(\Theta) = \frac{m^*_{SE_j}(\Theta)}{m^*_{SE_j}(\mathrm{AdP})+m^*_{SE_j}(\neg\mathrm{AdP})+m^*_{SE_j}(\Theta)}$$

where any $m_{SE_j}(\mathrm{AdP}), m_{SE_j}(\neg\mathrm{AdP}), m_{SE_j}(\Theta)$ may be ≤ 0. When this is the case m_{SE_j} is called a *pseudo belief function* [13].

In the same way we may calculate the support for decisive conditions, and the military end state.

For decisive conditions, we define

$$m_{DC_j}(\mathrm{AdP}) \triangleq m^{\alpha_{ji}}_{\oplus\{m_{SE_i}\}_{i=1}^n}(\mathrm{AdP})$$
$$= K\Big\{\prod_i \Big[1-\frac{\mathrm{impact}(SE_i, DC_j)}{10} m_{SE_i}(\neg\mathrm{AdP})\Big]$$
$$-\prod_i \Big[1-\frac{\mathrm{impact}(SE_i, DC_j)}{10} m_{SE_i}(\mathrm{AdP}) \tag{21}$$
$$-\frac{\mathrm{impact}(SE_i, DC_j)}{10} m_{SE_i}(\neg\mathrm{AdP})\Big]\Big\}$$

and

$$m_{DC_j}(\neg\mathrm{AdP}) \triangleq m^{\alpha_{ji}}_{\oplus\{m_{SE_i}\}_{i=1}^n}(\neg\mathrm{AdP})$$
$$= K\Big\{\prod_i \Big[1-\frac{\mathrm{impact}(SE_i, DC_j)}{10} m_{SE_i}(\mathrm{AdP})\Big]$$
$$-\prod_i \Big[1-\frac{\mathrm{impact}(SE_i, DC_j)}{10} m_{SE_i}(\mathrm{AdP}) \tag{22}$$
$$-\frac{\mathrm{impact}(SE_i, DC_j)}{10} m_{SE_i}(\neg\mathrm{AdP})\Big]\Big\}$$

with

$$m_{DC_j}(\Theta) \triangleq m^{\alpha_{ji}}_{\oplus\{m_{SE_i}\}_{i=1}^n}(\Theta)$$
$$= K\prod_i \Big[1-\frac{\mathrm{impact}(SE_i, DC_j)}{10} m_{SE_i}(\mathrm{AdP}) \tag{23}$$
$$-\frac{\mathrm{impact}(SE_i, DC_j)}{10} m_{SE_i}(\neg\mathrm{AdP})\Big]$$

As previously, we may manage the normalization by first calculating

$$m^*_{DC_j}(\mathrm{AdP}) = \frac{m_{DC_j}(\mathrm{AdP})}{K}$$
$$m^*_{DC_j}(\neg\mathrm{AdP}) = \frac{m_{DC_j}(\neg\mathrm{AdP})}{K} \tag{24}$$
$$m^*_{DC_j}(\Theta) = \frac{m_{DC_j}(\Theta)}{K}$$

using Eq. (21), Eq. (22), and Eq. (23), followed by calculating

$$
\begin{aligned}
m_{DC_j}(\mathrm{AdP}) &= \frac{m^*_{DC_j}(\mathrm{AdP})}{m^*_{DC_j}(\mathrm{AdP}) + m^*_{DC_j}(\neg\mathrm{AdP}) + m^*_{DC_j}(\Theta)} \\
m_{DC_j}(\neg\mathrm{AdP}) &= \frac{m^*_{DC_j}(\neg\mathrm{AdP})}{m^*_{DC_j}(\mathrm{AdP}) + m^*_{DC_j}(\neg\mathrm{AdP}) + m^*_{DC_j}(\Theta)} \\
m_{DC_j}(\Theta) &= \frac{m^*_{DC_j}(\Theta)}{m^*_{DC_j}(\mathrm{AdP}) + m^*_{DC_j}(\neg\mathrm{AdP}) + m^*_{DC_j}(\Theta)}
\end{aligned}
\tag{25}
$$

Similarly, for the military end state, we have

$$
\begin{aligned}
m_{MES}(\mathrm{AdP}) &\triangleq m^{\alpha_{ji}}_{\oplus\{m_{DC_i}\}_{i=1}^{n}}(\mathrm{AdP}) \\
&= K\left\{\prod_i\left[1 - \frac{\mathrm{impact}(DC_i, MES)}{10} m_{DC_i}(\neg\mathrm{AdP})\right]\right. \\
&\quad -\prod_i\left[1 - \frac{\mathrm{impact}(DC_i, MES)}{10} m_{DC_i}(\mathrm{AdP})\right. \\
&\quad \left.\left. - \frac{\mathrm{impact}(DC_i, MES)}{10} m_{DC_i}(\neg\mathrm{AdP})\right]\right\}
\end{aligned}
\tag{26}
$$

and

$$
\begin{aligned}
m_{MES}(\neg\mathrm{AdP}) &\triangleq m^{\alpha_{ji}}_{\oplus\{m_{DC_i}\}_{i=1}^{n}}(\neg\mathrm{AdP}) \\
&= K\left\{\prod_i\left[1 - \frac{\mathrm{impact}(DC_i, MES)}{10} m_{DC_i}(\mathrm{AdP})\right]\right. \\
&\quad -\prod_i\left[1 - \frac{\mathrm{impact}(DC_i, MES)}{10} m_{DC_i}(\mathrm{AdP})\right. \\
&\quad \left.\left. - \frac{\mathrm{impact}(DC_i, MES)}{10} m_{DC_i}(\neg\mathrm{AdP})\right]\right\}
\end{aligned}
\tag{27}
$$

with

$$
\begin{aligned}
m_{MES}(\Theta) &\triangleq m^{\alpha_{ji}}_{\oplus\{m_{DC_i}\}_{i=1}^{n}}(\Theta) \\
&= K\prod_i\left[1 - \frac{\mathrm{impact}(DC_i, MES)}{10} m_{DC_i}(\mathrm{AdP})\right. \\
&\quad \left. - \frac{\mathrm{impact}(DC_i, MES)}{10} m_{DC_i}(\neg\mathrm{AdP})\right]
\end{aligned}
\tag{28}
$$

As before, for simplicity, we may manage the normalization by first calculating

$$
\begin{aligned}
m^*_{MES}(\mathrm{AdP}) &= \frac{m_{MES}(\mathrm{AdP})}{K} \\
m^*_{MES}(\neg\mathrm{AdP}) &= \frac{m_{MES}(\neg\mathrm{AdP})}{K} \\
m^*_{MES}(\Theta) &= \frac{m_{MES}(\Theta)}{K}
\end{aligned}
\tag{29}
$$

using Eq. (26), Eq. (27), and Eq. (28), followed by calculating

$$
\begin{aligned}
m_{MES}(\mathrm{AdP}) &= \frac{m^*_{MES}(\mathrm{AdP})}{m^*_{MES}(\mathrm{AdP}) + m^*_{MES}(\neg\mathrm{AdP}) + m^*_{MES}(\Theta)} \\
m_{MES}(\neg\mathrm{AdP}) &= \frac{m^*_{MES}(\neg\mathrm{AdP})}{m^*_{MES}(\mathrm{AdP}) + m^*_{MES}(\neg\mathrm{AdP}) + m^*_{MES}(\Theta)} \\
m_{MES}(\Theta) &= \frac{m^*_{MES}(\Theta)}{m^*_{MES}(\mathrm{AdP}) + m^*_{MES}(\neg\mathrm{AdP}) + m^*_{MES}(\Theta)}
\end{aligned}
\tag{30}
$$

With these calculations we have all pieces of a subjective EBA algorithm (Algorithm 1).

Algorithm 1: Subjective EBA

- For all SE_j calculate:

  $m_{SE_j}(\mathrm{AdP})$, $m_{SE_j}(\neg\mathrm{AdP})$, $m_{SE_j}(\Theta)$ using Eq. (20);
- For all DC_j calculate:

  $m_{DC_j}(\mathrm{AdP})$, $m_{DC_j}(\neg\mathrm{AdP})$, $m_{DC_j}(\Theta)$ using Eq. (25);
- Calculate:

  $m_{MES}(\mathrm{AdP})$, $m_{MES}(\neg\mathrm{AdP})$, $m_{MES}(\Theta)$ using Eq. (30);
- Return all calculated values.

In Fig. 3 the calculated values of Algorithm 1 are presented in the upper part labelled "Effects", together with the initial subjective assessments $m_{A_j}(\mathrm{AdP})$, $m_{A_j}(\neg\mathrm{AdP})$, and $m_{A_j}(\Theta)$ in the lower part labelled "Activities" within the CSMT. Obviously, $m(\mathrm{AdP})$ is indicated by the green part, $m(\neg\mathrm{AdP})$ by the red part and the uncommitted $m(\Theta)$ by the gray part.

In order to further enhance the usability it may be of value to include a diagram of the change over time for these assessments. In Fig. 4 this is exemplified for the Military End State as calculated by Eq. (30) at different times.

4 Decomposing assessments

In this section we will decompose the belief functions (or pseudo belief functions), calculated in Algorithm 1 by Eq. (20), Eq. (25) and Eq. (30), into its separate components for and against the plan using the decomposition introduced by Smets [13], i.e.,

$$m_i(A) = 1 - \prod_{B \supseteq A} Q^{\alpha}_{\oplus\{m_i\}_{i=1}^{n}}(B)^{(-1)^{|B|-|A|+1}}. \tag{31}$$

This is done in order to observe the strength for and against the plan as if they are two separate pieces of evidence. Their combination is the result already calculated in Algorithm 1. Thus, decomposition is the inverse of combination.

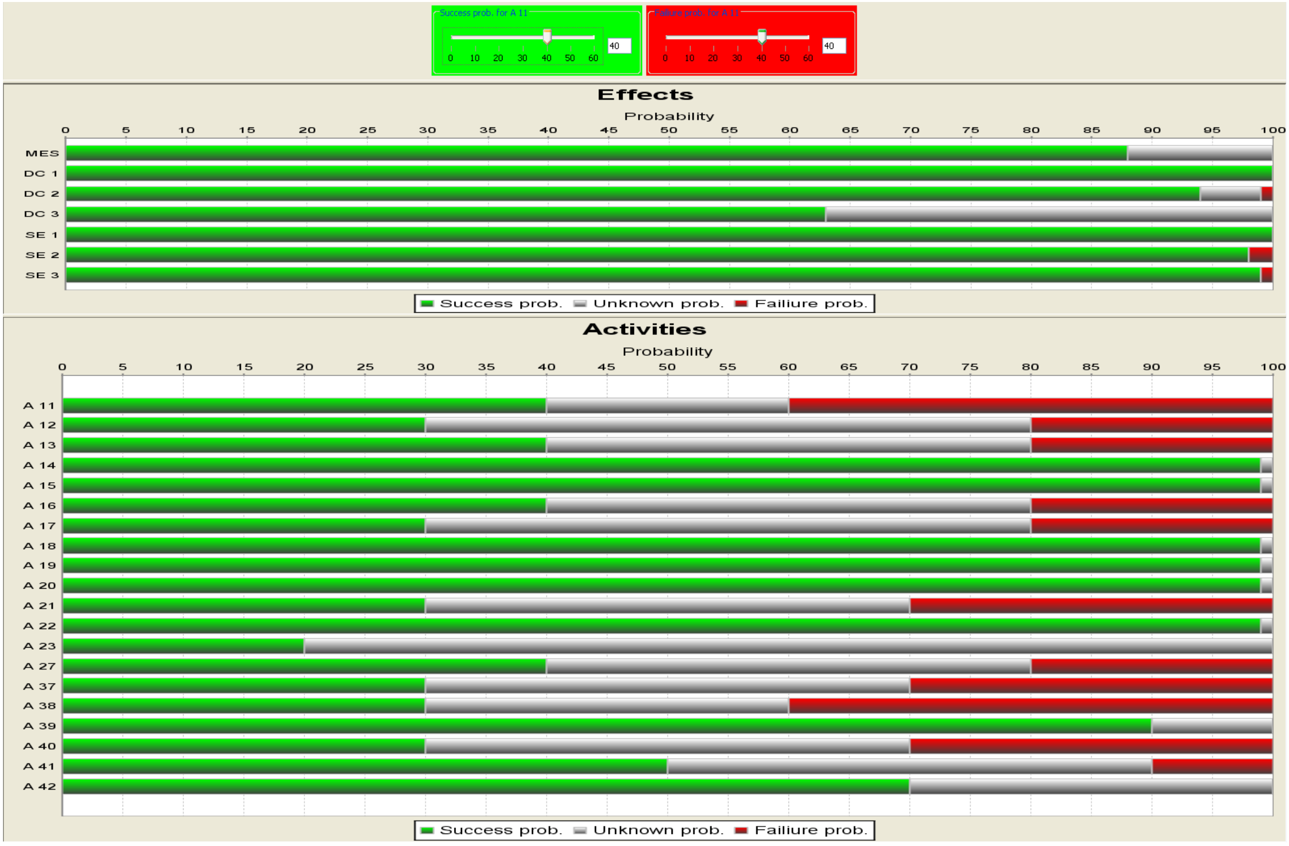

Fig. 3. Subjective Effect-Based Assessment (EBA) in the Collaborative Synchronization Management Tool (CSMT).

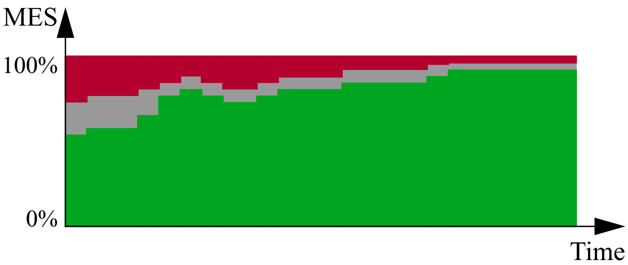

Fig. 4. Subjective assessments over time of Military End State.

First, we begin by calculating commonalities for all supporting effects using Eq. (4). We have

$$\begin{aligned} Q_{SE_j}(\mathrm{AdP}) &= m_{SE_j}(\mathrm{AdP}) + m_{SE_j}(\Theta) \\ Q_{SE_j}(\neg\mathrm{AdP}) &= m_{SE_j}(\neg\mathrm{AdP}) + m_{SE_j}(\Theta) \\ Q_{SE_j}(\Theta) &= m_{SE_j}(\Theta) \end{aligned} \tag{32}$$

where $m_{SE_j}(\mathrm{AdP})$, $m_{SE_j}(\neg\mathrm{AdP})$ and $m_{SE_j}(\Theta)$ are calculated using Eq. (20).

Secondly, using Eq. (31) and Eq. (32), we get

$$\begin{aligned} m^{+}_{SE_j}(\mathrm{AdP}) &= 1 - \frac{Q_{SE_j}(\Theta)}{Q_{SE_j}(\mathrm{AdP})} \\ m^{+}_{SE_j}(\Theta) &= 1 - Q_{SE_j}(\Theta) \end{aligned} \tag{33}$$

and

$$\begin{aligned} m^{-}_{SE_j}(\neg\mathrm{AdP}) &= 1 - \frac{Q_{SE_j}(\Theta)}{Q_{SE_j}(\neg\mathrm{AdP})} . \\ m^{-}_{SE_j}(\Theta) &= 1 - Q_{SE_j}(\Theta) \end{aligned} \tag{34}$$

where $m^{+}_{SE_j}$ and $m^{-}_{SE_j}$ are two different ISSFs, i.e., $m^{+}_{SE_j}(\mathrm{AdP}), m^{-}_{SE_j}(\neg\mathrm{AdP}) \in (-\infty, \infty)$, with

$$m_{SE_j} = m^{+}_{SE_j} \oplus m^{-}_{SE_j} . \tag{35}$$

Here Eq. (33) and Eq. (34) are the two sought after components bringing evidence for ($m^{+}_{SE_j}$) and against ($m^{-}_{SE_j}$) the plan, respectively.

If it turns out that $m^{+}_{SE_j}(\mathrm{AdP}) < 0$ or $m^{-}_{SE_j}(\neg\mathrm{AdP}) < 0$ then we may instead calculate the degree to which we do *not* believe in that proposition. When $m^{+}_{SE_j}(\mathrm{AdP}) < 0$ we have,

$$\begin{aligned} m^{+\mathrm{not}}_{SE_j}(\text{not believe in AdP}) &= 1 - \frac{1}{1 - m^{+}_{SE_j}(\mathrm{AdP})} \\ m^{+\mathrm{not}}_{SE_j}(\Theta) &= 1 - m^{+\mathrm{not}}_{SE_j}(\text{not believe in AdP}) \end{aligned} \tag{36}$$

where $m^{+\mathrm{not}}_{SE_j} \in [0, 1]$ is a SSF.

The support for "not believe in AdP" should be interpreted as the additional support needed before you are uncommitted towards AdP, i.e., $m^{+}_{SE_j}(\Theta) = 1$. Only when you receive more than $m^{+\mathrm{not}}_{SE_j}(\text{not believe in AdP})$ additional support for AdP should you start believing in it.

We should note that, usually

$$m^{+\text{not}}_{SE_j}(\text{not believe in AdP}) \neq m^{-}_{SE_j}(\neg\text{AdP}) \tag{37}$$

as the left hand side of Eq. (37) can be rewritten as

$$\begin{aligned} m^{+\text{not}}_{SE_j}(\text{not believe in AdP}) &= 1 - \frac{1}{1 - m^{+}_{SE_j}(\text{AdP})} \\ &= 1 - \frac{1}{1 - \left(1 - \frac{Q_{SE_j}(\Theta)}{Q_{SE_j}(\text{AdP})}\right)} \\ &= 1 - \frac{Q_{SE_j}(\text{AdP})}{Q_{SE_j}(\Theta)} \end{aligned} \tag{38}$$

which is in general different from the right hand side of Eq. (37),

$$m^{-}_{SE_j}(\neg\text{AdP}) = 1 - \frac{Q_{SE_j}(\Theta)}{Q_{SE_j}(\neg\text{AdP})}. \tag{39}$$

Similarly, when $m^{-}_{SE_j}(\neg\text{AdP}) < 0$ we have,

$$\begin{aligned} m^{-\text{not}}_{SE_j}(\text{not believe in } \neg\text{AdP}) &= 1 - \frac{1}{1 - m^{-}_{SE_j}(\text{AdP})} \\ m^{-\text{not}}_{SE_j}(\Theta) &= 1 - m^{-\text{not}}_{SE_j}(\text{not believe in } \neg\text{AdP}) \end{aligned} \tag{40}$$

where $m^{-\text{not}}_{SE_j} \in [0, 1]$ is also a SSF.

These two decomposed SSFs are then two independent subjective assessments for and against the successful outcome of SE_j, respectively.

We may perform the same type of decomposition for the Decisive Conditions. Using Eq. (4), we get

$$\begin{aligned} Q_{DC_j}(\text{AdP}) &= m_{DC_j}(\text{AdP}) + m_{DC_j}(\Theta) \\ Q_{DC_j}(\neg\text{AdP}) &= m_{DC_j}(\neg\text{AdP}) + m_{DC_j}(\Theta) \\ Q_{DC_j}(\Theta) &= m_{DC_j}(\Theta) \end{aligned} \tag{41}$$

where $m_{DC_j}(\text{AdP})$, $m_{DC_j}(\neg\text{AdP})$ and $m_{DC_j}(\Theta)$ can be calculated using Eq. (25).

Furthermore, using Eq. (31) and Eq. (41) we have

$$\begin{aligned} m^{+}_{DC_j}(\text{AdP}) &= 1 - \frac{Q_{DC_j}(\Theta)}{Q_{DC_j}(\text{AdP})} \\ m^{+}_{DC_j}(\Theta) &= 1 - Q_{DC_j}(\Theta) \end{aligned} \tag{42}$$

and

$$\begin{aligned} m^{-}_{DC_j}(\neg\text{AdP}) &= 1 - \frac{Q_{DC_j}(\Theta)}{Q_{DC_j}(\neg\text{AdP})}. \\ m^{-}_{DC_j}(\Theta) &= 1 - Q_{DC_j}(\Theta) \end{aligned} \tag{43}$$

When $m^{+}_{DC_j}(\text{AdP}) < 0$ we have,

$$\begin{aligned} m^{+\text{not}}_{DC_j}(\text{not believe in AdP}) &= 1 - \frac{1}{1 - m^{+}_{DC_j}(\text{AdP})} \\ m^{+\text{not}}_{DC_j}(\Theta) &= 1 - m^{+\text{not}}_{DC_j}(\text{not believe in AdP}) \end{aligned} \tag{44}$$

and when $m^{-}_{DC_j}(\neg\text{AdP}) < 0$ we have,

$$\begin{aligned} m^{-\text{not}}_{DC_j}(\text{not believe in } \neg\text{AdP}) &= 1 - \frac{1}{1 - m^{-}_{DC_j}(\text{AdP})} \\ m^{-\text{not}}_{DC_j}(\Theta) &= 1 - m^{-\text{not}}_{DC_j}(\text{not believe in } \neg\text{AdP}) \end{aligned} \tag{45}$$

where $m^{+\text{not}}_{DC_j}$ and $m^{-\text{not}}_{DC_j}$ are SSFs.

Finally, we perform a single decomposition for the Military End State, again using Eq. (4). We get,

$$\begin{aligned} Q_{MES}(\text{AdP}) &= m_{MES}(\text{AdP}) + m_{MES}(\Theta) \\ Q_{MES}(\neg\text{AdP}) &= m_{MES}(\neg\text{AdP}) + m_{MES}(\Theta) \\ Q_{MES}(\Theta) &= m_{MES}(\Theta) \end{aligned} \tag{46}$$

where $m_{MES}(\text{AdP})$, $m_{MES}(\neg\text{AdP})$ and $m_{MES}(\Theta)$ can be calculated using Eq. (30).

In the same manner as before we get, using Eq. (31) and Eq. (46),

$$\begin{aligned} m^{+}_{MES}(\text{AdP}) &= 1 - \frac{Q_{MES}(\Theta)}{Q_{MES}(\text{AdP})} \\ m^{+}_{MES}(\Theta) &= 1 - Q_{MES}(\Theta) \end{aligned} \tag{47}$$

and

$$\begin{aligned} m^{-}_{MES}(\neg\text{AdP}) &= 1 - \frac{Q_{MES}(\Theta)}{Q_{MES}(\neg\text{AdP})}. \\ m^{-}_{MES}(\Theta) &= 1 - Q_{MES}(\Theta) \end{aligned} \tag{48}$$

If $m^{+}_{MES}(\text{AdP}) < 0$ then we calculate support in

$$\begin{aligned} m^{+\text{not}}_{MES}(\text{not believe in AdP}) &= 1 - \frac{1}{1 - m^{+}_{MES}(\text{AdP})} \\ m^{+\text{not}}_{MES}(\Theta) &= 1 - m^{+\text{not}}_{MES}(\text{not believe in AdP}) \end{aligned} \tag{49}$$

and if $m^{-}_{MES}(\neg\text{AdP}) < 0$ we calculate

$$\begin{aligned} m^{-\text{not}}_{MES}(\text{not believe in } \neg\text{AdP}) &= 1 - \frac{1}{1 - m^{-}_{MES}(\text{AdP})} \\ m^{-\text{not}}_{MES}(\Theta) &= 1 - m^{-\text{not}}_{MES}(\text{not believe in } \neg\text{AdP}) \end{aligned} \tag{50}$$

where $m^{+\text{not}}_{MES}$ and $m^{-\text{not}}_{MES}$ are SSFs.

As before these propositions should be interpreted as the additional belief needed before we are completely uncommitted towards these propositions, i.e., before $m^{+\text{not}}_{MES}(\Theta) = 1$ and $m^{-\text{not}}_{MES}(\Theta) = 1$, respectively.

These equations, Eq. (47) and Eq. (48), are the two parts expressing support for and against the overall plan. Together with their combination

$$m_{MES} = m^{+\text{not}}_{MES} \oplus m^{-\text{not}}_{MES}, \tag{51}$$

already calculated directly by Eq. (30), they express the best subjective assessment on the outcome of the plan.

All of this can be put together into Algorithm 2 for calculating subjective EBA with decomposed support for and against each elements of the overall plan.

Performing and updating Algorithm 2 whenever new subjective assessments regarding activities are received, should give planners the earliest indication on future outcomes of all plan elements and any opportunity to replan during execution when necessary.

Algorithm 2: Subjective EBA with decomposition

- For all SE_j calculate:

  $m_{SE_j}(\text{AdP})$, $m_{SE_j}(\neg\text{AdP})$, $m_{SE_j}(\Theta)$ using Eq. (20);

  $m^{+}_{SE_j}(\text{AdP})$, $m^{+}_{SE_j}(\Theta)$, using Eq. (33);

  $m^{-}_{SE_j}(\neg\text{AdP})$, $m^{-}_{SE_j}(\Theta)$ using Eq. (34);

  If $m^{+}_{SE_j}(\text{AdP}) < 0$ calculate:

  $m^{+\text{not}}_{SE_j}(\text{not believe in AdP})$, $m^{+\text{not}}_{SE_j}(\Theta)$ using Eq. (36);

  If $m^{-}_{SE_j}(\neg\text{AdP}) < 0$ calculate:

  $m^{-\text{not}}_{SE_j}(\text{not believe in }\neg\text{AdP})$, $m^{+\text{not}}_{SE_j}(\Theta)$ using Eq. (40);

- For all DC_j calculate:

  $m_{DC_j}(\text{AdP})$, $m_{DC_j}(\neg\text{AdP})$, $m_{DC_j}(\Theta)$ using Eq. (25);

  $m^{+}_{DC_j}(\text{AdP})$, $m^{+}_{DC_j}(\Theta)$, using Eq. (42);

  $m^{-}_{DC_j}(\neg\text{AdP})$, $m^{-}_{DC_j}(\Theta)$ using Eq. (43);

  If $m^{+}_{DC_j}(\text{AdP}) < 0$ calculate:

  $m^{+\text{not}}_{DC_j}(\text{not believe in AdP})$, $m^{+\text{not}}_{DC_j}(\Theta)$ using Eq. (44);

  If $m^{-}_{DC_j}(\neg\text{AdP}) < 0$ calculate:

  $m^{-\text{not}}_{DC_j}(\text{not believe in }\neg\text{AdP})$, $m^{+\text{not}}_{DC_j}(\Theta)$ using Eq. (45);

- Calculate $m_{MES}(\text{AdP})$, $m_{MES}(\neg\text{AdP})$, and $m_{MES}(\Theta)$ using Eq. (30).

  $m^{+}_{MES}(\text{AdP})$, $m^{+}_{MES}(\Theta)$, using Eq. (47);

  $m^{-}_{MES}(\neg\text{AdP})$, $m^{-}_{MES}(\Theta)$ using Eq. (48);

  If $m^{+}_{MES}(\text{AdP}) < 0$ calculate:

  $m^{+\text{not}}_{MES}(\text{not believe in AdP})$, $m^{+\text{not}}_{MES}(\Theta)$ using Eq. (49);

  If $m^{-}_{MES}(\neg\text{AdP}) < 0$ calculate:

  $m^{-\text{not}}_{MES}(\text{not believe in }\neg\text{AdP})$, $m^{+\text{not}}_{MES}(\Theta)$ using Eq. (50);

- Return all calculated values.

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5 Conclusions

We have developed a subjective Effects-Based Assessment method for making subjective assessment of plans and plan elements within the Effects-Based Approach to Operations.

We have shown that such subjective assessments can be performed of all supporting effects, decisive conditions and the military end state by taking human subjective assessments about activities as input and extending those assessments to all other plan elements using a cross impact matrix.

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