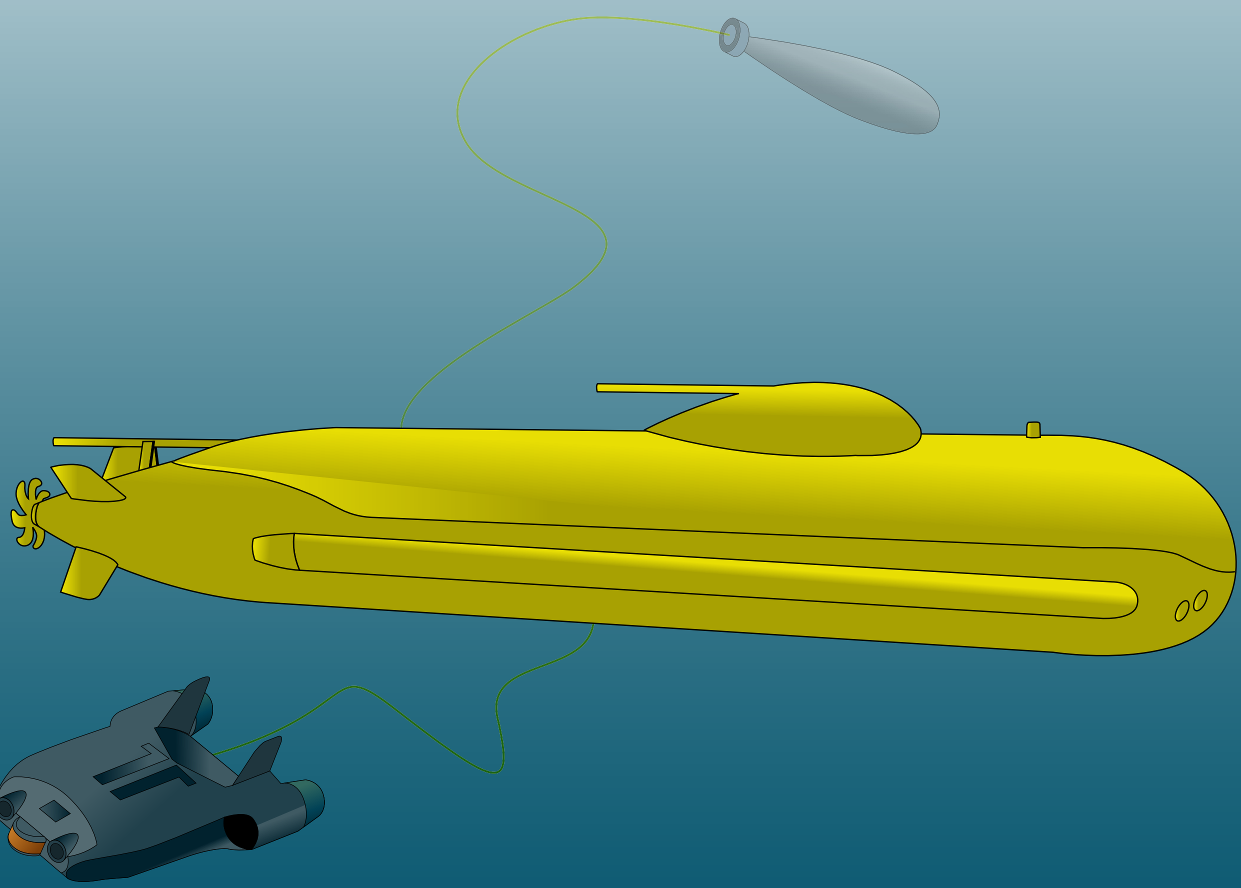
FOI-R--1469--SE
December 2004
1650-1942

Technical report

Johan Hamberg
Anders Lennartsson

Manoeuvering of underwater vehicles



Systems Technology Division
SE-172 90 STOCKHOLM
Sweden

Weapons and Protection Division
SE-147 25 TUMBA
Sweden

Swedish Defence Research Agency
Weapons and Protection Division
SE-147 25 TUMBA
Sweden

FOI-R--1469--SE
December 2004
1650-1942

Technical report

Johan Hamberg
Anders Lennartsson

Manoeuvering of underwater vehicles

Issuing organization	Report number, ISRN	Report type
Swedish Defence Research Agency Weapons and Protection Division SE-147 25 TUMBA Sweden	FOI-R--1469--SE	Technical report
	Research area code Strike and Protection	
	Month year December 2004	Project no. E2029
	Subcategory Underwater Weapons	
	Subcategory 2	
Author/s (editor/s) Johan Hamberg Anders Lennartsson	Project manager Niklas Alin	
	Approved by Monica Dahlén	
	Sponsoring agency Swedish Armed Forces	
	Scientifically and technically responsible Martin Hagström	

Report title

Manoeuvering of underwater vehicles

Abstract

A basis for controlling high performance underwater vehicles is suitable control systems. One key factor for producing such control systems is the availability of detailed models of dynamics of submerged bodies. This report presents an improved model of this kind, in which more complicated bodies than earlier can be described, for example vehicle driven with water-jets. In addition some aspects of modular control system architectures for unmanned vehicles are discussed.

Keywords

Control, underwater, vehicles, dynamics, added mass, Kirchhoff

Further bibliographic information	Language
	English
ISSN 1650-1942	Pages 17
Distribution By sendlist	Price Acc. to pricelist Security classification Unclassified

Utgivare	Rapportnummer, ISRN	Klassificering
Totalförsvarets forskningsinstitut Avdelningen för Vapen och skydd SE-147 25 TUMBA Sweden	FOI-R--1469--SE	Teknisk rapport
	Forskningsområde: Bekämpning och skydd	
	Månad, år: December 2004	Projektnummer: E2029
	Delområde: Undervattensvapen	
	Delområde 2:	

Författare/redaktör	
Johan Hamberg Anders Lennartsson	Projektledare: Niklas Alin
	Godkänd av: Monica Dahlén
	Uppdragsgivare/kundbeteckning: Försvarsmakten
	Tekniskt och/eller vetenskapligt ansvarig: Martin Hagström

Rapportens titel

Manövrering av undervattensfarkoster

Sammanfattning

För design av styrlagar och styrautomater för högpresterande undervattensfarkoster behövs noggranna modeller av dynamiken hos sådana farkoster. Denna rapport beskriver en förbättrad modell för detta fall där också farkoster som drivs med till exempel vattenjet kan beskrivas. Dessutom beskrivs översiktligt modulära styrsystem för obemannade undervattensfarkoster.

Nyckelord

Manövrering, undervattensfarkoster, dynamik, manöverderivator, Kirchhoff

Övriga bibliografiska uppgifter	Språk
	Engelska
ISSN: 1650-1942	Antal sidor: 17
Distribution: Enligt missiv	Pris: Enligt prislista Sekretess: Öppen

Contents

1. Introduction

Underwater vehicle control is a broad subject with a long history, since the first underwater vehicles were developed in the second half of the 19th century. Over the past few years work has been made in this area within the project *Precision engagement under water*, in Swedish *Flexibla verkansystem under vatten*. Focus has been on studying techniques that enable better control of underwater vehicles, in order to enhance performance for increased accuracy and agility. Obviously such techniques are important in underwater warfare. Two important ingredients in such work are better models that incorporate more detailed dynamics, and control system architectures, including guidance principles and control laws, that take advantage of these more detailed models. Additionally, detailed models form a basis for tight integration of control and navigation systems, which promises to enhance accuracy. This latter issue is important for controllable weapons effects that is likely to be a tactical requirement by the Swedish Armed Forces within a few years.

Underwater vehicles operating at moderate speeds are adequately described by so called Kirchhoff's equations, whereby inifinite dimensional dynamical system consisting of the combined fluid and body motions is reduced to a finit dimensional dynamical system. This report presents some novel results regarding these issues, in particular with respect to detailed models of underwater dynamics, where Kirchhoff's equations are generalized in three directions. In parallel, and partly studied in other projects, our work on and knowledge of nonlinear controller design has also progressed. Such methods are now a viable way for design of control laws. Thus, detailed models of dynamic properties that are suitably adapted to the design process are now becomming essential to obtain control systems of high performance, and it is not a coincidence that the methods applied in the work on modelling, as well as study of control system architectures, are well adapted to these nonlinear design methods.

The first sections describe in some detail the dynamics of submerged bodies with displacing volumes that possibly contain holes, which in mathematical terms is phrased *not necessarily simply connected*, after which follows a section with a brief introduction to modular control systems. Modularity of software, as well as electronic and mechanical hardware, is clearly related to *standardization*. Clever use of standardization within projects has the potential of reducing costs and extending the feasible life time of the project outcome. From a researcher's viewpoint it is mainly the flexibility that is appealing, but the prospect of reducing life-cycle costs suggests that the subject is important by itself.

2. Underwater vehicle dynamics

The dynamics of submerged vehicles is described by Kirchhoff's equations. In this chapter a derivation of these equations is given, closely following the presentation in [6]. The discussion is then generalised to deformable submerged bodies with nontrivial topology.

2.1 The Lie Group $SE(3)$

The Lie Group $E(3)$ consists of the isometries of Euclidean space $\mathbb{R}^3$. An element of $E(3)$ has the form

$$\kappa : \boldsymbol{x} \mapsto \boldsymbol{R}\boldsymbol{x} + \boldsymbol{b} \tag{2.1}$$

where $\boldsymbol{R}$ is an orthonormal 3×3 matrix and $\boldsymbol{b} \in \mathbb{R}^3$. With the identification $\boldsymbol{x} \longleftrightarrow \begin{pmatrix} \boldsymbol{x} & 1 \end{pmatrix}^T \in \mathbb{R}^4$, κ is represented by the multiplication by the matrix

$$\mathcal{R} = \begin{pmatrix} \boldsymbol{R} & \boldsymbol{b} \\ 0 & 1 \end{pmatrix} \tag{2.2}$$

and $E(3)$ is identified with the corresponding subgroup of $GL(4)$. It is clear that the determinant of an element of $E(3)$ equals $+1$ or -1. The identity component of $E(3)$ consists of elements with positive determinant, it is denoted by $SE(3)$.

The Lie algebra $se(3)$ of $SE(3)$ is then identified with the subset of $gl(4)$ consisting of matrices of the form

$$\boldsymbol{\Omega} = \begin{pmatrix} \boldsymbol{\omega}\times & \boldsymbol{u} \\ 0 & 0 \end{pmatrix} \tag{2.3}$$

where $\boldsymbol{\omega}\times$ is the skew-symmetric matrix

$$\boldsymbol{\omega}\times = \begin{pmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{pmatrix}$$

associated to the vector $\boldsymbol{\omega} = (\omega_1, \omega_2, \omega_3)$.

In $se(3)$ the Lie bracket is given by the matrix commutator (as inherited from $gl(4)$)

$$[\boldsymbol{\Omega}_A, \boldsymbol{\Omega}_B] = \boldsymbol{\Omega}_A\boldsymbol{\Omega}_B - \boldsymbol{\Omega}_B\boldsymbol{\Omega}_A = \begin{pmatrix} \boldsymbol{\omega}_C\times & \boldsymbol{u}_C \\ 0 & 0 \end{pmatrix}$$

where $\boldsymbol{\omega}_C = \boldsymbol{\omega}_A \times \boldsymbol{\omega}_B$ and $\boldsymbol{u}_C = \boldsymbol{\omega}_A \times \boldsymbol{u}_B - \boldsymbol{\omega}_B \times \boldsymbol{u}_A$

The *Adjoint representation* of $E(3)$ on $se(3)$ is the mapping

$$\boldsymbol{\Omega} \mapsto Ad_{\mathcal{R}}\boldsymbol{\Omega} = \begin{pmatrix} \boldsymbol{R} & \boldsymbol{b} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \boldsymbol{\omega}\times & \boldsymbol{u} \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \boldsymbol{R} & \boldsymbol{b} \\ 0 & 1 \end{pmatrix}^{-1}$$

which may be written as

$$\begin{pmatrix} \boldsymbol{\omega} \\ \boldsymbol{u} \end{pmatrix} \mapsto \begin{pmatrix} \det(\boldsymbol{R})\boldsymbol{R} & \boldsymbol{0} \\ \det(\boldsymbol{R})\,(\boldsymbol{b}\times)\,\boldsymbol{R} & \boldsymbol{R} \end{pmatrix} \begin{pmatrix} \boldsymbol{\omega} \\ \boldsymbol{u} \end{pmatrix}$$

i.e.

$$Ad_{\mathcal{R}} = \begin{pmatrix} \det(\boldsymbol{R})\boldsymbol{R} & \boldsymbol{0} \\ \det(\boldsymbol{R})\,(\boldsymbol{b}\times)\,\boldsymbol{R} & \boldsymbol{R} \end{pmatrix}$$

and the corresponding Lie algebra representation (the *adjoint representation* on $se(3)$) is hence given by

$$ad_{\boldsymbol{\Omega}} = \begin{pmatrix} \boldsymbol{\omega}\times & \boldsymbol{0} \\ \boldsymbol{u}\times & \boldsymbol{\omega}\times \end{pmatrix}$$

Similarly the *coadjoint representation* (on $se(3)^*$) is given by

$$ad^*_{\boldsymbol{\Omega}} = \begin{pmatrix} \boldsymbol{\omega}\times & \boldsymbol{u}\times \\ \boldsymbol{0} & \boldsymbol{\omega}\times \end{pmatrix}$$

Consider now a motion $t \mapsto (\boldsymbol{R}(t), \boldsymbol{b}(t)) = \mathcal{R}(t)$. The velocity parameters (linear and angular velocity) in the body frame $\boldsymbol{\Omega}$ and in the spatial frame $\tilde{\boldsymbol{\Omega}}$ are then given by

$$\frac{d}{dt}\mathcal{R} = \tilde{\boldsymbol{\Omega}}\mathcal{R} = \mathcal{R}\boldsymbol{\Omega}$$

By means of the $Ad_{\mathcal{R}}$-operation, we obtain a handy formalism for changing between the velocity parameters in the body frame, $\boldsymbol{\Omega}$, and in the spatial frame $\tilde{\boldsymbol{\Omega}} = Ad_{\mathcal{R}}\boldsymbol{\Omega}$ and for reexpressing the velocity parameters with respect to a new body origin $\boldsymbol{r}_A$, which is given by the formula $Ad_{(\boldsymbol{1},\boldsymbol{r}_A)}\boldsymbol{\Omega}$.

2.2 The dynamics of perfect fluids

Euler's equations for a homogeneous ideal impressible fluid of density ρ are

$$\rho\left(\partial_t \boldsymbol{v} + (\boldsymbol{v}\cdot\nabla)\,\boldsymbol{v}\right) = -\nabla\left(p + \rho U\right) \quad (2.4)$$

$$\nabla\cdot\boldsymbol{v} = 0 \quad (2.5)$$

where U is a potential for the gravity field. The pressure is *a priori* undetermined, but is determined by solving (2.4) under the kinematic constraint (2.5) together with appropriate boundary conditions.

Euler's equations (2.4) may be rewritten as

$$\partial_t \boldsymbol{v} + \nabla\left(\frac{1}{2}\boldsymbol{v}\cdot\boldsymbol{v} + p/\rho + U\right) = \boldsymbol{v}\times(\nabla\times\boldsymbol{v}) \quad (2.6)$$

By taking the curl of (2.6) one obtains that $\boldsymbol{w} = \nabla\times\boldsymbol{v}$ satisfies $\partial_t\boldsymbol{w} = \nabla\times(\boldsymbol{v}\times\boldsymbol{w})$, so if $\boldsymbol{w}\equiv 0$ at $t = 0$, then $\boldsymbol{w}\equiv 0$ at all t. In all that follows we restrict our attention to the case where $\boldsymbol{w}$ vanishes identically. This is necessary for the reduction to a finite dimensional system. Furthermore, in the presence of an otherwise negligible viscosity, the vorticity will asymptotically approach zero over time. Assuming that the container is simply connected, we may then write the velocity field of the fluid as

$$\boldsymbol{v} = \nabla\phi \tag{2.7}$$

and (for suitably chosen ϕ) Euler's equations (2.4) and (2.5) take the forms

$$\partial_t\phi + \frac{1}{2}\nabla\phi\cdot\nabla\phi + p/\rho + U = \mathbf{0} \tag{2.8}$$

$$\Delta\phi = 0 \tag{2.9}$$

Now, consider a system formed by a perfect fluid and a submerged ("solid") body $\mathbb{B}$ which may be deformable, but such that the perfect slipping boundary conditions hold:

$$(\boldsymbol{v} - \tilde{\boldsymbol{v}}_{solid})\cdot\boldsymbol{n} = 0$$

where $\boldsymbol{v}$ and $\tilde{\boldsymbol{v}}_{solid}$ are the velocities of the fluid and $\mathbb{B}$ at a point of contact and $\boldsymbol{n}$ is the unit normal vector of the solid/fluid interface $\partial\mathbb{B}$ (oriented in outwards from $\mathbb{B}$). Assuming that the exterior of the solid is simply connected, the velocity is given by (2.7) where ϕ is the (unique) solution to the exterior Neumann boundary value problem

$$\begin{aligned}\Delta\phi &= 0 \text{ in } \mathbb{R}^3\backslash\mathbb{B}\\ \frac{\partial\phi}{\partial n} &= \tilde{\boldsymbol{v}}_{solid}\cdot\boldsymbol{n} \text{ on } \partial\mathbb{B}\\ \phi &\to 0 \text{ as } |\boldsymbol{r}|\to\infty\end{aligned}$$

When ϕ is known, the pressure at $\partial\mathbb{B}$ may be reconstructed by (2.8).

Let $\psi_{\boldsymbol{r}_1}(\boldsymbol{r}) = \frac{1}{4\pi|\boldsymbol{r}-\boldsymbol{r}_1|}$. It follows from Green's formula that

$$\int_{\partial\mathbb{B}}\left(\psi_{\boldsymbol{r}_1}(\boldsymbol{r})\frac{\partial\phi(\boldsymbol{r})}{\partial n_{\boldsymbol{r}}} - \phi(\boldsymbol{r})\frac{\partial\psi_{\boldsymbol{r}_1}(\boldsymbol{r})}{\partial n_{\boldsymbol{r}}}\right)dA_{\boldsymbol{r}} = \frac{1}{2}\phi(\boldsymbol{r}_1) \quad \forall\boldsymbol{r}_1 \in \partial\mathbb{B}$$

whenever $\Delta\phi = 0$ in $\mathbb{R}^3\backslash\mathbb{B}$. If we insert our boundary conditions into this equation, we obtain an integral equation for determining ϕ (on $\partial\mathbb{B}$).

$$\int_{\partial\mathbb{B}}\left(\psi_{\boldsymbol{r}_1}(\boldsymbol{r})\tilde{\boldsymbol{v}}_{solid}\cdot\boldsymbol{n} - \phi(\boldsymbol{r})\frac{\partial\psi_{\boldsymbol{r}_1}(\boldsymbol{r})}{\partial n_{\boldsymbol{r}}}\right)dA_{\boldsymbol{r}} = \frac{1}{2}\phi(\boldsymbol{r}_1) \tag{2.10}$$

When ϕ is known (from solving(2.10)) on $\partial\mathbb{B}$, it may be utilised to determine ϕ in $\mathbb{R}^3\backslash\mathbb{B}$ by another instance of Green's formula

$$\int_{\partial\mathbb{B}}\left(\psi_{\boldsymbol{r}_1}(\boldsymbol{r})\tilde{\boldsymbol{v}}_{solid}\cdot\boldsymbol{n} - \phi(\boldsymbol{r})\frac{\partial\psi_{\boldsymbol{r}_1}(\boldsymbol{r})}{\partial n_{\boldsymbol{r}}}\right)dA_{\boldsymbol{r}} = \phi(\boldsymbol{r}_1) \quad \forall\boldsymbol{r}_1 \in \text{interior}\left(\mathbb{R}^3\backslash\mathbb{B}\right)$$

However, we will not need the latter expression, since the quantity of immediate interest, *viz.* the kinetic energy of the fluid, may be reexpressed in terms of quantities defined on $\partial\mathbb{B}$ only:

$$\int_{fluid} \frac{1}{2}\boldsymbol{v}^2 dm = \frac{\rho}{2}\int_{\mathbb{R}^3\backslash\mathbb{B}} (\nabla\phi)^2\, dV = \frac{\rho}{2}\int_{\mathbb{R}^3\backslash\mathbb{B}} \nabla\cdot(\phi\nabla\phi)\, dV = \frac{-\rho}{2}\int_{\partial\mathbb{B}} \phi\tilde{\boldsymbol{v}}_{solid}\cdot\boldsymbol{n} dA \tag{2.11}$$

In particular, when $\mathbb{B}$ is rigid, $\tilde{\boldsymbol{v}}_{solid}\cdot\boldsymbol{n}$ becomes a linear expression in $\begin{pmatrix}\boldsymbol{\omega}\\ \boldsymbol{v}_O\end{pmatrix}$ and the fluid's kinetic energy is consequently a quadratic form in $\begin{pmatrix}\boldsymbol{\omega}\\ \boldsymbol{v}_O\end{pmatrix}$, the explicit coefficients of which may be computed by solving 6 integral equations of the form (2.10) and computing 21 numbers of the form $\int_{\partial\mathbb{B}}\phi_A\frac{\partial\phi_B}{\partial n}dA$.

2.3 The total kinetic energy

If $\mathbb{B}$ in addition to its "rigid motion degrees of freedom" has certain shape degrees of freedom $y = (y_1, ..y_m)$, such as rudder angles, the kinetic energy becomes a quadratic form in $(\boldsymbol{\omega}, \boldsymbol{v}_0, \dot{y})$, depending parametrically on y. This obviously also holds for total kinetic energy of the body $\mathbb{B}$ and the fluid.

In case of a rigid body $\mathbb{B}$ the total kinetic energy is thus of the form

$$T = \frac{1}{2}\Omega^T\mathbb{J}\Omega = \frac{1}{2}\begin{pmatrix}\boldsymbol{\omega}\\ \boldsymbol{v}_O\end{pmatrix}^T\begin{pmatrix}\boldsymbol{J} & \boldsymbol{D}\\ \boldsymbol{D} & \boldsymbol{M}\end{pmatrix}\begin{pmatrix}\boldsymbol{\omega}\\ \boldsymbol{v}_O\end{pmatrix}$$

As is shown in [6], a generalised Steiner's formula, the formula for changing reference point (from O to A) is given by

$$\mathbb{J}_A = \left(Ad_{(\mathbf{1},\boldsymbol{r}_A)}\right)^T\mathbb{J}_O Ad_{(\mathbf{1},\boldsymbol{r}_A)}$$

and if

$$\mathcal{Q} = \begin{pmatrix}\boldsymbol{Q} & \mathbf{0}\\ 0 & 1\end{pmatrix}$$

is a *symmetry* of the system in the sense that it leaves invariant both the interior mass distribution in $\mathbb{B}$ and the exterior contour $\partial\mathbb{B}$, it holds that

$$\mathbb{J}_O = (Ad_{\mathcal{Q}})^T\mathbb{J}_O Ad_{\mathcal{Q}} \tag{2.12}$$

From the formula (2.12) it follows that if reflection in the plane $(O, \boldsymbol{e}_3)$ is a symmetry, then

$$\begin{aligned}
\boldsymbol{J}_O &= \begin{pmatrix} J_{11} & J_{12} & 0\\ J_{12} & J_{22} & 0\\ 0 & 0 & J_{33}\end{pmatrix}\\
\boldsymbol{M}_O &= \begin{pmatrix} M_{11} & M_{12} & 0\\ M_{12} & M_{22} & 0\\ 0 & 0 & J_{33}\end{pmatrix}\\
\boldsymbol{D}_O &= \begin{pmatrix} 0 & 0 & D_{13}\\ 0 & 0 & D_{23}\\ D_{31} & D_{32} & 0\end{pmatrix}
\end{aligned}$$

and that if rotation an angle $2\pi/k$ ($k \geq 3$) around the axis $(O, \boldsymbol{e}_3)$ is a symmetry, then

$$\begin{aligned}
\boldsymbol{J}_O &= \begin{pmatrix} J_1 & 0 & 0 \\ 0 & J_1 & 0 \\ 0 & 0 & J_3 \end{pmatrix} \\
\boldsymbol{M}_O &= \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_1 & 0 \\ 0 & 0 & M_3 \end{pmatrix} \\
\boldsymbol{D}_O &= \begin{pmatrix} D_1 & -\delta & 0 \\ \delta & D_1 & 0 \\ 0 & 0 & D_3 \end{pmatrix}
\end{aligned}$$

It follows that for a highly symmetric cigar-shaped torpedo

$$\begin{aligned}
\boldsymbol{J}_O &= \begin{pmatrix} J_1 & 0 & 0 \\ 0 & J_1 & 0 \\ 0 & 0 & J_3 \end{pmatrix} \\
\boldsymbol{M}_O &= \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_1 & 0 \\ 0 & 0 & M_3 \end{pmatrix} \\
\boldsymbol{D}_O &= \boldsymbol{0}
\end{aligned}$$

when the rudder angles are zero.

2.4 Kirchhoff's equations and generalisations

It can be shown [7], [4] that the dynamics of the fluid+body system in the vorticity free case is given by Lagrange's equations on $SE(3)$. Furthermore, the kinetic energy is left invariant on $SE(3)$, so the dynamical system simplifies into one defined on $se(3)$, given by the Kirchhoff's equations.

$$\frac{d}{dt}\left(\mathbb{J}\boldsymbol{\Omega}\right) + ad^*_{\boldsymbol{\Omega}}\left(\mathbb{J}\boldsymbol{\Omega}\right) = \mathbb{M} \tag{2.13}$$

where

$$\begin{aligned}
\mathbb{J} &= \begin{pmatrix} \boldsymbol{J} & \boldsymbol{D} \\ \boldsymbol{D}^T & \boldsymbol{M} \end{pmatrix} \\
\boldsymbol{\Omega} &= \begin{pmatrix} \boldsymbol{\omega} \\ \boldsymbol{v}_O \end{pmatrix} \\
\mathbb{M} &= \begin{pmatrix} \boldsymbol{M}_O \\ \boldsymbol{F} \end{pmatrix}
\end{aligned}$$

as above, and $\mathbb{M}$ consists of the moment sum $\boldsymbol{M}_O$ (w.r.t. O) and force sum $\boldsymbol{F}$ of the exterior forces acting on the body (including net buoyancy forces and torques). Recall also that

$$ad^*_{\boldsymbol{\Omega}}\left(\begin{pmatrix} \boldsymbol{L}_O \\ \boldsymbol{p} \end{pmatrix}\right) = \begin{pmatrix} \boldsymbol{\omega}\times & \boldsymbol{v}_O\times \\ \boldsymbol{0} & \boldsymbol{\omega}\times \end{pmatrix}\begin{pmatrix} \boldsymbol{L}_O \\ \boldsymbol{p} \end{pmatrix}$$

Now the same arguments that are used for deriving Kirchhoff's equations from first principles may in fact be used to generalise these in three directions.

- First, we may allow topologically more complicated bodies, such that the fluid occupies a multiply connected domain. Then a vorticity free fluid velocity field no longer needs to have a single valued velocity potential. It can however be shown (essentially by Hodge theory) that a vorticity free field may be decomposed into one term with a globally defied velocity potential and another term which is orthogonal to all potential flows and which, when unactuated, does not affect the solid body's motion. The latter term is therefore ignored in the present report.

- Second, a fluid together with a deformable submerged body (with a finite number of degrees of freedom) still satisfies Lagrange's equations (*cf.* [8]), which in the more general case take the form

$$\begin{aligned} \frac{d}{dt}\left(\frac{\partial T}{\partial \boldsymbol{\Omega}}\right) + ad^*_{\boldsymbol{\Omega}}\left(\frac{\partial T}{\partial \boldsymbol{\Omega}}\right) &= \mathbb{M} \\ \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{y}}\right) - \frac{\partial T}{\partial y} &= Q_y \end{aligned}$$

  where $T(\boldsymbol{\Omega}, y, \dot{y})$ is the total kinetic energy and Q_y is the generalised force covector associated with the y-variables (rudder moments etc.).

- Third, the assumptions of an infinite fluid surrounding which was used in the derivation of (2.10) and (2.11) may be relaxed. If the appropriate boundary conditions at the fluids interface to e.g. a container is taken into account, a similar Lagrangian formulation of the dynamics is possible. The equations will then, however, depend strongly on the position of $\mathbb{B}$. In the case of a flat seabed, for instance, the boundary conditions lead to a "ground effect" equivalent to as if there were no seabed, but instead a mirror image body present. This generalisation will not be discussed further in the present report though.

As a final remark on the Kirchhoff's equations (with or without a fluid) is that the choice of reduction point O is completely arbitrary, and that the right hand side component $\boldsymbol{M}_O$ is indeed the moment sum without any "correction terms", in contrast with most other formulations of the moment equation w.r.t. general comoving reduction points.

3. Control principles for underwater vehicles

In the following questions of stability, stabilisation and control of Kirchhoff's equations will be addressed.

3.1 Stability

What are the stationary states of the Kirchhoff dynamics, *i.e.* which are the equilibrium points of the $se(3)$ dynamical system and to what motions do these correspond? A stationary point $\boldsymbol{\Omega}_0 = \begin{pmatrix} \boldsymbol{\omega}_0 \\ \boldsymbol{v}_0 \end{pmatrix}$ is such that (2.13) is satisfied with $\mathbb{M} = 0$ and $\dot{\boldsymbol{\Omega}}_0 = 0$. This means that

$$\begin{pmatrix} \boldsymbol{\omega}_0 \times & \boldsymbol{v}_0 \times \\ \mathbf{0} & \boldsymbol{\omega}_0 \times \end{pmatrix} \begin{pmatrix} \boldsymbol{J} & \boldsymbol{D} \\ \boldsymbol{D}^T & \boldsymbol{M} \end{pmatrix} \begin{pmatrix} \boldsymbol{\omega}_0 \\ \boldsymbol{v}_0 \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \end{pmatrix} \tag{3.1}$$

that is

$$\begin{pmatrix} \boldsymbol{\omega}_0 \times (\boldsymbol{J}\boldsymbol{\omega}_0 + \boldsymbol{D}\boldsymbol{v}_0) + \boldsymbol{v}_0 \times \left(\boldsymbol{D}^T\boldsymbol{\omega}_0 + \boldsymbol{M}\boldsymbol{v}_0\right) \\ \boldsymbol{\omega}_0 \times \left(\boldsymbol{D}^T\boldsymbol{\omega}_0 + \boldsymbol{M}\boldsymbol{v}_0\right) \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \end{pmatrix} \tag{3.2}$$

The condition (3.2_2), $\boldsymbol{\omega}_0 \times \left(\boldsymbol{D}^T\boldsymbol{\omega}_0 + \boldsymbol{M}\boldsymbol{v}_0\right) = \mathbf{0}$, leaves us with two possibilities:

- $\boldsymbol{\omega}_0 = 0$, *i.e.* a pure translational velocity field
- $\boldsymbol{\omega}_0 \neq 0$, in which case $\boldsymbol{D}^T\boldsymbol{\omega}_0 + \boldsymbol{M}\boldsymbol{v}_0 = \lambda\boldsymbol{\omega}_0$, for some real λ, in other words,
$$\boldsymbol{v}_0 = \boldsymbol{M}^{-1}\left(\lambda\boldsymbol{I} - \boldsymbol{D}^{\boldsymbol{T}}\right)\boldsymbol{\omega}_0$$

In the former case (pure translation), the condition (3.2_1) reads $\boldsymbol{v}_0 \times \boldsymbol{M}\boldsymbol{v}_0 = \mathbf{0}$**,** so either $\boldsymbol{v}_0 = 0$ or $\boldsymbol{v}_0$ is an eigenvector to $\boldsymbol{M}$. In the $\boldsymbol{\omega}_0 \neq 0$ case, the condition (3.2_1) becomes

$$\boldsymbol{\omega}_0 \times \left(\boldsymbol{J} + (\boldsymbol{D} - \boldsymbol{\lambda}\boldsymbol{I})\boldsymbol{M}^{-1}\left(\lambda\boldsymbol{I} - \boldsymbol{D}^{\boldsymbol{T}}\right)\right)\boldsymbol{\omega}_0 = \mathbf{0}$$

so $\boldsymbol{\omega}_0$ is an eigenvector to $\boldsymbol{J} + (\boldsymbol{D} - \boldsymbol{\lambda}\boldsymbol{I})\boldsymbol{M}^{-1}\left(\lambda\boldsymbol{I} - \boldsymbol{D}^{\boldsymbol{T}}\right)$.

This means that, generically when eigenvalues are simple, there are three distinct one-dimensional families of stationary points $\boldsymbol{\Omega}_0$, each one parametrised by the parameter λ, and such that

- $\boldsymbol{\omega}_0$ is the k:th eigenvector to $\boldsymbol{J} + (\boldsymbol{D} - \boldsymbol{\lambda}\boldsymbol{I})\boldsymbol{M}^{-1}\left(\lambda\boldsymbol{I} - \boldsymbol{D}^{\boldsymbol{T}}\right)$, $(k = 1, 2, 3)$
- $\boldsymbol{v}_0 = \boldsymbol{M}^{-1}\left(\lambda\boldsymbol{I} - \boldsymbol{D}^{\boldsymbol{T}}\right)\boldsymbol{\omega}_0$

These eigenmodes of motion may be considered as "motion primitives" from which more complicated motions may be built up.

For each of these possible equilibria, the corresponding stability properties may be assessed by considering the linearised system.

If for instance

$$\begin{aligned} \boldsymbol{J}_O &= \begin{pmatrix} J_1 & 0 & 0 \\ 0 & J_1 & 0 \\ 0 & 0 & J_3 \end{pmatrix} \\ \boldsymbol{M}_O &= \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_1 & 0 \\ 0 & 0 & M_3 \end{pmatrix} \\ \boldsymbol{D}_O &= \mathbf{0} \\ \boldsymbol{\omega}_0 &= \mathbf{0} \\ \boldsymbol{v}_0 &= \begin{pmatrix} 0 \\ 0 \\ V \end{pmatrix} \end{aligned} \tag{3.3}$$

the linearised dynamics is given by

$$\frac{d}{dt}\begin{pmatrix} \delta\omega_1 \\ \delta\omega_2 \\ \delta\omega_3 \\ \delta v_1 \\ \delta v_2 \\ \delta v_3 \end{pmatrix} = V \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{M_1+M_3}{J1} & 0 \\ 0 & 0 & 0 & -\frac{M_1+M_3}{J1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{M_3}{M_1} & 0 & 0 & 0 & 0 \\ -\frac{M_3}{M_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \delta\omega_1 \\ \delta\omega_2 \\ \delta\omega_3 \\ \delta v_1 \\ \delta v_2 \\ \delta v_3 \end{pmatrix}$$

which has three double eigenvalues, 0 and $\pm iV\sqrt{\frac{M_3(M_1+M_3)}{M_1 J_1}}$. The two zero eigenvalues correspond to the eigenmodes of translation along/rotation around the symmetry axis $(O, \boldsymbol{e}_3)$. The nonzero eigenvalues correspond to coupled lateral rotational and translational ondulatory motions; the eigenvectors are

$$\begin{pmatrix} \pm i\sqrt{\frac{M_1(M_1+M_3)}{M_3 J_1}} & 0 & 0 & 0 & 1 & 0 \end{pmatrix}^T \tag{3.4}$$

and

$$\begin{pmatrix} 0 & \pm i\sqrt{\frac{M_1(M_1+M_3)}{M_3 J_1}} & 0 & 1 & 0 & 0 \end{pmatrix}^T \tag{3.5}$$

These modes of motion are similar to that known under the name "Dutch roll" in aerodynamics.

The *linearised* motion is qualitatively the same as that of a similarly shaped body moving freely in space (no surrounding fluid). The linearised dynamics around this equilibrium is stable, but not asymptotically stable. The nonlinear dynamics is however unstable.

We also note certain expressions that are constants of motion for the free motion, here expressed in terms of $\boldsymbol{L}_O$ and $\boldsymbol{p}$ given by

$$\begin{pmatrix} \boldsymbol{L}_O \\ \boldsymbol{p} \end{pmatrix} = \begin{pmatrix} \boldsymbol{J} & \boldsymbol{D} \\ \boldsymbol{D}^T & \boldsymbol{M} \end{pmatrix} \begin{pmatrix} \boldsymbol{\omega} \\ \boldsymbol{v}_O \end{pmatrix}$$

These expressions are given by

$$\begin{aligned} C_0 &= \frac{1}{2}\begin{pmatrix} \boldsymbol{L}_O \\ \boldsymbol{p} \end{pmatrix}^T \begin{pmatrix} \boldsymbol{J} & \boldsymbol{D} \\ \boldsymbol{D}^T & \boldsymbol{M} \end{pmatrix}^{-1} \begin{pmatrix} \boldsymbol{L}_O \\ \boldsymbol{p} \end{pmatrix} \\ C_1 &= \frac{1}{2}\boldsymbol{p}\cdot\boldsymbol{p} \\ C_2 &= \boldsymbol{L}_O\cdot\boldsymbol{p} \end{aligned}$$

The motion constant C_0 is of course the energy, while $\dot{C}_1 = \boldsymbol{p}\cdot\dot{\boldsymbol{p}} = \boldsymbol{p}\cdot(-\boldsymbol{\omega}\times\boldsymbol{p}) = 0$ and $\dot{C}_2 = \dot{\boldsymbol{L}}_O\cdot\boldsymbol{p} + \boldsymbol{L}_O\cdot\dot{\boldsymbol{p}} = (-\boldsymbol{\omega}\times\boldsymbol{L}_O - \boldsymbol{v}_O\times\boldsymbol{p})\cdot\boldsymbol{p} + \boldsymbol{L}_O\cdot(-\boldsymbol{\omega}\times\boldsymbol{p}) = 0$ are further scalar constants of motion related to the symmetry group. These constants of motion of course delimit the ways the system can exhibit instability.

3.2 Stabilisation

In this report only some general principles of stabilisation will be discussed. In order to stabilise an eigenmode some form of actuation is necessary. In the formulation suggested in the present report, all such actuation are in fact *interior*, in the sense that they involve the extra coordinates y mentioned above. This can be achieved in two essentially distinct ways:

- By means of interior mobile parts within the vehicle's hull. Movable masses may be used to – "statically" – adjust the position of the centre of mass or the moments of inertia. Flywheels may be used for a more dynamical adjustment of the body's inertial properties, *e.g.* stabilising unstable rotational modes. This method of stabilisation is utilised in [6].
- By means of mobile parts in contact with the fluid, which thereby changes the fluid boundary value problem, may be used for the more traditional vehicle control. Rudders, pumps and propellers belong to this family of actuators.

In the case of autonomous underwater vehicles, it seems safe to say that a combination of these types of actuators gives the best means of actuation.

Consider again the motion of a vehicle with extra degrees of freedom

$$\begin{aligned} \frac{d}{dt}\left(\frac{\partial T}{\partial \boldsymbol{\Omega}}\right) + ad^*_{\boldsymbol{\Omega}}\left(\frac{\partial T}{\partial \boldsymbol{\Omega}}\right) &= \mathbb{M} \\ \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{y}}\right) - \frac{\partial T}{\partial y} &= Q_y \end{aligned}$$

where $T(\boldsymbol{\Omega}, y, \dot{y})$ is the total kinetic energy and Q_y is the generalised force covector associated with the y-variables. In principle the unactuated part of the equations also involves derivatives $\dot{y}$ and $\ddot{y}$ of the auxiliary variables. In the case of rudders, this would allow for using these for propulsion (swimming). In practice, rudders would not be used in that way; this effect is negligible. Ignoring this minuscule effect amounts to changing the dynamical equations for the vehicle into

$$\mathbb{J}\dot{\boldsymbol{\Omega}} + ad^*_{\boldsymbol{\Omega}}(\mathbb{J}\boldsymbol{\Omega}) = \mathbb{M}$$

which now involve the y only "parametrically". The residual wrench $\mathbb{M}$ may contain unmodelled dynamics due to viscosity effect etc. In the following discussion it is ignored. Furthermore, in the dynamical equation $\dot{\mathbf{\Omega}} = -\mathbb{J}^{-1} ad^*_{\mathbf{\Omega}} (\mathbb{J}\mathbf{\Omega})$, we linearise in the control parameters y. If

$$\mathbb{J}(y) = \mathbb{J}(y_0) + \mathbb{J}'(y_0)\,\delta y + o(\delta y) \equiv \mathbb{J}_0 + \mathbb{J}_1 \delta y + o(\delta y)$$

we may consider the system

$$\dot{\mathbf{\Omega}} = -\mathbb{J}_O^{-1} ad^*_{\mathbf{\Omega}} (\mathbb{J}_O \mathbf{\Omega}) + \mathbb{J}_O^{-1} \left[\mathbb{J}_1 \mathbb{J}_O^{-1}, ad^*_{\mathbf{\Omega}}\right] (\mathbb{J}_O \mathbf{\Omega})\,\delta y$$

or, put differently

$$\mathbb{J}_0 \dot{\mathbf{\Omega}} + ad^*_{\mathbf{\Omega}} (\mathbb{J}_O \mathbf{\Omega}) = \left[\mathbb{J}_1 \mathbb{J}_O^{-1}, ad^*_{\mathbf{\Omega}}\right] (\mathbb{J}_O \mathbf{\Omega})\,\delta y \tag{3.6}$$

It may be clear from physical principles that the vehicle is "practically controllable" with very few independent actuators (rudders). This however involves genuinely nonlinear effects. The linearised version of (3.6) in the case (3.3) needs no less than three independent rudder actuators to become linearly controllable.

In that case the linearised system has the standard form

$$\dot{x} = Ax + Bu$$

where the state variable x is the deviation from the commanded velocity, A is the matrix $-\mathbb{J}_O^{-1} ad^*_{\mathbf{\Omega}} \mathbb{J}_O$, u is the commanded rudder angle deviations δy, and the columns of B are given by $\left[\mathbb{J}_1 \mathbb{J}_O^{-1}, ad^*_{\mathbf{\Omega}}\right] (\mathbb{J}_O \mathbf{\Omega})$ with different $\mathbb{J}_1$ for different rudders. In this case $A^k B = 0$ for $k \geq 2$, and the different $\mathbb{J}_1$ must be such that the corresponding B satisfies

$$\operatorname{rank}(B, AB) = 6$$

and stabilisation by means of for instance pole placement becomes a standard procedure.

One possible "generic" method of steering the vehicle is by concatenating the equilibrium motion primitives, stabilised by the above method. When changing from one motion primitive to another, the old equilibrium is made unstable, after which the new desired equilibrium is made stable. If the manoeuvre is slow enough compared to the (linear) dynamics time constants, the motion may be made one of sliding along a trajectory of equilibria, in a orderly fashion. This is the technique of gain scheduling.

Other generic control strategies include the use of interior flywheels for stabilising certain motion primitives, and using the rudders to manoeuvre between these.

4. Modular control system for underwater vehicles

This section contains a review of requirements on control systems for underwater vehicles in particular, although much of it is applicable to other types of vehicles. Differences in dynamic behaviour between vehicles for different areas of operation, be it flying, rolling or swimming, comes in mainly in the actual control law modules that are part of a functioning control system. Future system may also have dual behaviour which necessitates the inclusion of modules with widely spread functionality.

Two strong motivations for modular control systems for autonomous vehicles are the possibility for re-use of design work and code, and secondly that different modes of control will probably be called for during different phases of a system's operational service. If a control system for a particular piece of hardware is modular by providing well designed logical interfaces and control functions, the hardware can more easily be used together with other systems with control systems that can interact with the particular controller. Similarly, if several different controllers exist for some hardware, and all are easily exchanged or modified due to requirements set by the current users, the control system in itself is modular. One example of the latter is to use different controllers during training and real operations, another is the use of different controllers for different functionality during distinctive phases of a single mission.

Further subdividing the operational envelope and creating many controllers where each is useable in a small patch may be thought of as a special case of the general idea of modularity. The latter method of designing controllers are used in real systems, and called *gain scheduling*, and shall not be treated in more detail here. Instead we are more interested in large scale features of control system architectures.

4.1 Control principles

Control principles for unmanned vehicles can be grouped into:

- Interactive control
- Automatic deterministic control
- Automatic reactive control

Interactive control is synonymous with remote control. Automatic or autonomous control is divided into the two groups *automatic deterministic control* which handles noise and model errors, and *automatic reactive control* which is more capable of handling unexpected events.

With interactively controlled systems, the operator need not necessarily be a human. As long as the specifications and details of the interface for controlling the system are followed, the controlling system can be an automatic or autonomous system by itself. Control system architectures that allow such interchange of controllers, selecting between physical persons and control modules that are exercising remote or on-board control are clearly at an advantage when it comes to flexiblity and adaptivity. Architectures of this type provide for stacking and layering of autonomous and non-autonomous systems and must be considered a requirement for enabling the full potential when using a mix of systems for complex tasks.

4.2 Requirements on control system architectures

Some reasonable requirements on software systems for automatic and autonomous functions in unmanned vehicles include:

- Allow control law modules that are based on different control paradigms.
- It must be possible to implement relevant parts according to standards for security-critical software, e.g. DO-178B [12] [3] or IEC 61508 [5].
- The system permits simple changes of sensor and actuator configurations.
- The system simplifies the use of identical code in physical platforms as well as simulations.
- The system integrates well with other software developed or accessible by the developers and users.
- Useful on different types of platforms and vehicles, for instance autonomous underwater vehicles, AUV, but also surface and air vehicles.
- The system allows operator or operators to control all or parts of the vehicle and subsystems, independently of level of autonomy.
- Facilitate cooperation between different vehicles and other systems. Necessitates the capability for communication and opens up the, yet unmapped, area of algorithms and methods for implementing collaborative behaviour.

Most control systems developed today are implemented as software. Developing safety critical software systems poses special requirements for the desing and implementation phase, code revision and control etc. This is however not covered here.

4.3 Control system architectures

A survey of some relevant systems has been made to identify those that at least partially fulfill the requirements above, or are intended to fit a similar role as our yet immature system. The findings were as follows.

4.3.1 AVEC The system *Autonomous VEhicle Control* (AVEC) is developed at FOI. It is aimed at providing flexible software for control of mobile robots. The requirements described earlier in section 4.2 were partly derived as a result of the work on AVEC.

4.3.2 J-UCAS COS Common Operating System, COS, is a project within the joint unmanned combat aer system, J-UCAS, run by Darpa, US Air Force, and US Navy. COS is intended to facilitate cooperation between several aerial vehicles, ground control stations and other involved actors. The system will be used on the test aircrafts X-45 and X-47. On the web page [2] it is stated:

The COS will enable interoperability among multiple air vehicles and control stations, facilitating the integration of other subsystems such as sensors, weapons, and communications. The COS encompasses the software architecture, algorithms, applications and services that provide command and control, communications management, mission planning, much of the interactive autonomy, the human systems interface and the many other qualities associated with the J-UCAS system. The J-UCAS system architecture will ensure intra-operability between the internal components of J-UCAS and inter-operability with external elements such as manned aircraft, command and control centers, and space assets.

4.3.3 ORCA ORCA [10] is a set of software tools for developing component based robotic systems. It is an extension of the project *Open Robot Control Software*, see section 4.3.4.

4.3.4 OROCOS Open Robot Control Software [11] (OROCOS), has one more name, *Open Realtime Control Service*.

4.3.5 MARS *Mobile Autonomous Robot Software* (MARS) [9] was a project for developing complete, effective and higly adaptable software for cooperating autonomous robots. It was finished during the fall of 2004. During the project several robots and control systems were developed, in particular for technology for soccer-playing robots.

4.3.6 CARMEN *Carnegie Mellon Robot Navigation Toolkit* (CARMEN) [1] is a project aimed at developing open software for control of mobile robots. It is a part of the MARS project described in 4.3.5.

4.4 Evaluating system architectures

Obviously, the immediate needs of the users impact the decision of selecting a system architecture for programming autonomous behaviour in. A checklist based on a specification of the requirements listed by the potential users is crucial during this phase. The following list of important points is derived from our work with requirements for a modular control system to be used in our research vehicles.

- Support for real-time operations?

- Abstractions for sensors and actuators?
- Abstractions for data and data-flows from sensors?
- Multi-domain system, for water, ground and air?
- How is events and callbacks from sensors and actuators handled?
- Licensing issues?
- Configurability?
- What computer hardware is necessary?
- What operating systems are supported?
- Is there active development, maintenance and a large user group?
- How good is the documentation?
- Is the developing group interested in partnerships around their software?

One final issue is whether the system takes advantage of current standards and best practices, for instance in digital communication.

4.5 Vehicles and other hardware

Currently, the vehicles and platforms considered for research within our group is a small UGV, a small torpedoe, and the PLUMS ROV, a remotely operated submersible with a tether for power and control, is used to carry instruments for various kinds of experiments. At present the local control system onboard PLUMS is run on a microcontroller. The aim is to implement control modules that mimic the current system and allows the user interface to be essentially the same, while at the same time provide for future upgrades of better controllability and autonomy of certain functions that reduces operator workload.

Bibliography

[1] Carnegie Mellon Robot Navigation Toolkit.
http://www-2.cs.cmu.edu/~carmen/.

[2] DARPA J-UCAS programme. http://www.darpa.mil/j-ucas/fact_sheet.htm, http://www.darpa.mil/baa/PS04-04.html.

[3] FAA Federal Aviation Administration. RTCA, Inc., Document RTCA/DO-178B. Advisory Circular AC20-115B, http://www.faa.gov/.

[4] Thor I. Fossen. *Guidance and Control of Ocean Vehicles*. John Wiley and Sons, 1994.

[5] IEC. Functional safety of electrical/electronic/programmable electronic safetyrelated systems. IEC 61508, http://www.iec.ch/.

[6] Krister Jacobsson. Modeling of a generic underwater vehicle with mathmodelica. Scientific Report FOI-R--0537--SE, FOI, SE-172 90 Stockholm, June 2002.

[7] Scott D. Kelly. *The Mechnics and Control of Robotic Locomotion with Applications to Aquatic Vehicles*. PhD thesis, California Institute of Technology, Pasadena, California, 1998.

[8] R. Mason and J. Burdick. Propulsion and control of deformable bodies in an ideal fluid. In *Proceedings of the 1999 International Symposium on Robotics and Automation*, 1999.

[9] Mobile Autonomous Robot Software.
http://www-2.cs.cmu.edu/~multirobotlab/MARS/.

[10] Open source framework for developing component-based robotic systems.
http://orca-robotics.sourceforge.net/.

[11] Open Robot Control Software.
http://www.orocos.org/.

[12] Software considerations in airborne systems and equipment certification. RTCA/DO-178B, http://www.rtca.org/.